A new connectivity model for Cognitive Radio Ad-Hoc Networks: definition and exploiting for routing design

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To those who believe in me...
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Introduction

During my Phd I studied Cognitive Radio Ad-Hoc Networks (CRAHNs), focusing my attention on connectivity and routing design.

The cognitive radio paradigm has been proposed as a technique for solving the spectrum shortage problem. This technology enables cognitive devices to sense the spectrum and detect available spectrum holes to be used for transmission. Spectrum holes are defined as a set of frequency bands temporarily unoccupied by their licensed users (called Primary Users-PUs). The cognitive device should be able to access these spectrum bands without interfering and impacting on the PUs. Typically, the cognitive devices constitute a Secondary Users (SUs) underlay network where the SUs cooperate in a distributed fashion to achieve network connectivity and communication by using free licensed spectrum bands. When the cognitive devices are organized in a distributed multi-hop architecture, with dynamic network topology and with time and location varying spectrum availability, the secondary network is also CRAHN.

In traditional wireless ad-hoc networks nodes communicate by using the same frequency: the distance among nodes and the adopted transmission power are the main parameters affecting the connectivity of the network topology. Oppositely, in a CRAHN SUs experience spectrum heterogeneity, because the set of available spectrum bands might be quite different from node to node and might dynamically change over time and location due to PUs’ activities. Consequently, in a CRAHN two nodes can connect if they are in radio visibility and have at least one available spectrum bands in common. For these reason, not only the nodes position and transmission power, but also their free spectrum holes affect network connectivity. The concept of connectivity has to be revised and worked up, in order to take into account the presence
and the behavior of PUs. These aspects have a significant impact on communication reliability as well as on the design of routing schemes. In particular, the routing procedures need of cross-layer approaches, since the cooperation between routing and spectrum management functionalities allows to take decision being aware of the surrounding spectrum environment.

These considerations motivate me to propose and implement a mathematical framework to evaluate the connectivity in a cognitive scenario. To this aim I used graph theory, where the network connectivity is evaluated by mean of the second smallest eigenvalue of the Laplacian matrix, named algebraic connectivity. I extended the definition of this parameter in order to consider the presence and the mean activity of PUs and I modeled the cognitive network with a cognitive aleatory graph. In this context the algebraic connectivity becomes an aleatory function of the PUs’ behavior and I provided a method to calculate its expected value. In this way it is possible to compare different networks from connectivity point of view. Besides I indicated a method that gives a good approximation of the expected value of the algebraic connectivity, reducing the computational complexity.

The proposed mathematical framework has been exploited to routing purpose: the idea is to use the revised concept of connectivity to compare stability of different paths. In a cognitive network, where the topology dynamically changes with PUs’ behavior, a path that is always the best one does not exist: in fact, each time a PU activates, can happen that the used path becomes out of order and consequently it is necessary to re-route. Re-routing each time a PU becomes active again is onerous, since signaling information has to be exchanged among SUs. So it could be more convenient to select a path that is not continuously interrupted by the appearance of PUs, accepting a little performance degradation. For this reason, I proposed a routing scheme, named Gymkhana, that aims at routing data packets across paths that avoid obstacles, that are network zones that do not guarantee stable and highly connectivity.

My thesis is organized into five Chapters. In Chapter 1 I introduced Cognitive Radio Networks (CRNs), focusing on their main functionalities and challenges: I described the Cognitive cycle, explaining the spectrum sensing, spectrum decision, spectrum sharing and spectrum mobility, and finally
I pointed out problems related to Common Control Channel in CRNs. In Chapter 2 I analyzed how the connectivity changes in this kind of networks, and I investigated how the revised concept of connectivity impacts on routing design. Besides I described the most important related works about connectivity and routing. In Chapter 3 I presented the proposed mathematical framework and the approximated method to evaluate the cognitive connectivity and I explained why and when the approximation is valid. In chapter 4 I presented the Gymkhana routing describing its components and evaluating its performance, both from a topological point of view and from a traffic perspective. Finally in Chapter 5 I conclude my thesis, outlining conclusions and future aspects.
Chapter 1

Cognitive Radio Networks

1.1 Cognitive Radio Networks

Today's wireless networks are regulated by a fixed spectrum assignment policy, i.e., the spectrum is regulated by governmental agencies and is assigned to license holders or services on a long term basis for large geographical regions. In addition, a large portion of the assigned spectrum is used sporadically as illustrated in Figure 1.1, where the signal strength distribution over a large portion of the wireless spectrum is shown. The spectrum usage is concentrated on certain portions of the spectrum, while a significant amount of the spectrum remains unused. According to Federal Communications Commission (FCC) [1], temporal and geographical variations in the utilization of the assigned spectrum range from 15% to 85%. Although the fixed spectrum assignment policy generally served well in the past, there is a dramatic increase in the access to the limited spectrum for mobile services in the recent years. This increase is straining the effectiveness of the traditional spectrum policies.

The limited available spectrum and the inefficiency in the spectrum usage necessitate a new communication paradigm to exploit the existing wireless spectrum opportunistically [2]. Dynamic spectrum access is proposed to solve these current spectrum inefficiency problems. NeXt Generation (xG) communication networks [3], also known as Dynamic Spectrum Access Networks (DSANs) as well as Cognitive Radio Networks (CRNs), will provide high
bandwidth to mobile users via heterogeneous wireless architectures and dynamic spectrum access techniques ([4]). The inefficient usage of the existing spectrum can be improved through opportunistic access to the licensed bands without interfering with the existing users. xG networks, however, impose several research challenges due to the broad range of available spectrum as well as diverse Quality-of-Service (QoS) requirements of applications. These heterogeneities must be captured and handled dynamically as mobile terminals roam between wireless architectures and along the available spectrum pool. The key enabling technology of xG networks is the cognitive radio. Cognitive radio techniques provide the capability to use or share the spectrum in an opportunistic manner. Dynamic spectrum access techniques allow the cognitive radio to operate in the best available channel. More specifically, the cognitive radio technology will enable the users to (1) determine which portions of the spectrum is available and detect the presence of licensed users when a user operates in a licensed band (spectrum sensing), (2) select the best available channel (spectrum management), (3) coordinate access to this channel with other users (spectrum sharing), and (4) vacate the channel when
a licensed user is detected (*spectrum mobility*). Once a cognitive radio supports the capability to select the best available channel, the next challenge is to make the network protocols adaptive to the available spectrum. Hence, new functionalities are required in an xG network to support this adaptivity. These functionalities of xG networks enable spectrum-aware communication protocols. However, the dynamic use of the spectrum causes adverse effects on the performance of conventional communication protocols, which were developed considering a fixed frequency band for communication. So far, networking in xG networks is an unexplored topic. The xG network communication components and their interactions are illustrated in Figure 1.2. It is evident from the significant number of interactions that the xG network functionalities necessitate a cross-layer design approach. More specifically, spectrum sensing and spectrum sharing cooperate with each other to enhance spectrum efficiency. In spectrum management and spectrum mobility functions, application, transport, routing, medium access and physical layer functionalities are carried out in a cooperative way, considering the dynamic nature of the underlying spectrum.

The basic idea of CRNs is that the unlicensed devices (also called cognitive radio users or Secondary Users-SUs) need to vacate the band once the licensed device (also known as a Primary User-PU) is detected. CR networks, however, impose unique challenges due to the high fluctuation in the available spectrum as well as diverse Quality of Service (QoS) requirements. Cognitive radio technology is the key technology that enables a CRAHN to use spectrum in a dynamic manner. The term, cognitive radio, can formally be defined as follows [5]: *a Cognitive Radio is a radio that can change its transmitter parameters based on interaction with the environment in which it operates.* From this definition, two main characteristics of the cognitive radio can be defined as follows [6],[7]:

- **Cognitive capability:** Cognitive capability refers to the ability of the radio technology to capture or sense the information from its radio environment. This capability cannot simply be realized by monitoring the power in some frequency bands of interest, but more sophisticated techniques, such as autonomous learning and action decision, are required in order to capture the temporal and spatial variations in the
radio environment and avoid interference to other users. Through this capability, the portions of the spectrum that are unused at a specific time or location can be identified. Consequently, the best spectrum and appropriate operating parameters can be selected.

• **Reconfigurability**: The cognitive capability provides spectrum awareness, whereas reconfigurability enables the radio to be dynamically programmed according to the radio environment. More specifically, the cognitive radio can be programmed to transmit and receive on a variety of frequencies and to use different transmission access technologies supported by its hardware design [8].

The ultimate objective of the cognitive radio is to obtain the best available spectrum through cognitive capability and reconfigurability as described before. Since most of the spectrum is already assigned, the most important challenge is to share the licensed spectrum without interfering with the transmission of other licensed users as illustrated in Figure 1.3. The cognitive radio enables the usage of temporarily unused spectrum, which is referred
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Figure 1.3: Spectrum Hole concept

to as spectrum hole or white space. If this band is further utilized by a licensed user, the cognitive radio moves to another spectrum hole or stays in the same band, altering its transmission power level or modulation scheme to avoid interference.

1.2 Cognitive Radio ad-hoc Networks and its differences with classical ad-hoc Networks

According to the network architecture, CRNs can be classified as infrastructure-based CRN and Cognitive Radio ad-hoc Networks (CRAHNs) [9]. The infrastructure-based CRN has a central network entity such as a base station in cellular networks or an access point in wireless local area networks (LANs). On the other hand, the CRAHN does not have any infrastructure backbone. Thus, a CR user can communicate with other CR users through ad hoc connection on both licensed and unlicensed spectrum bands. In the infrastructure-based CRNs, the observations and analysis performed by each
Figure 1.4: Comparison between CR capabilities for: (a) infrastructure-based CRNs, and (b) CRAHNs.

CR user feeds the central CR base-station, so that it can make decisions on how to avoid interfering with primary networks. According to this decision, each CR user reconfigures its communication parameters, as shown in Figure 1.4(a). On the contrary, in CRAHNs, each user needs to have all CR capabilities and is responsible for determining its actions based on the local observation, as shown in Figure 1.4(b). Since the CR user cannot predict the influence of its actions on the entire network with its local observation, cooperation schemes are essential, where the observed information can be exchanged among devices to broaden the knowledge on the network.

The changing spectrum environment and the importance of protecting the transmission of the licensed users of the spectrum mainly differentiate classical ad hoc networks from CRAHNs. The main differences are the following:

- *Choice of transmission spectrum*: In CRAHNs, the available spectrum bands are distributed over a wide frequency range, which vary over time and space. Thus, each user shows different spectrum availability according to PU activity. As opposed to this, classical ad hoc networks generally operate on a pre-decided channel that remains unchanged with time. For the ad hoc networks with multi-channel support, all the channels are continuously available for transmission, though nodes may select few of the latter from this set based on self-interference
constraints. A key distinguishing factor is the primary consideration of protecting the PU transmission, which is entirely missing in classical ad hoc networks.

- **Topology control**: Ad hoc networks lack centralized support, and hence must rely on local coordination to gather topology information. In classical ad hoc networks, this is easily accomplished by periodic beacon messages on the channel. However, in CRAHNs, as the licensed spectrum opportunity exists over a large range of frequencies, sending beacons over all the possible channels is not feasible. Thus, CRAHNs are highly probable to have incomplete topology information, which leads to an increase in collisions among CR users as well as interference to the PUs.

- **Multi-hop/multi-spectrum transmission**: The end-to-end route in the CRAHN consists of multiple hops having different channels according to the spectrum availability. Thus, CRAHNs require collaboration between routing and spectrum allocation in establishing these routes. Moreover, the spectrum switches on the links are frequent based on PU arrivals. As opposed to classical ad hoc networks, maintaining end-to-end QoS involves not only the traffic load, but also how many different channels and possibly spectrum bands are used in the path, the number of PU induced spectrum change events, consideration of periodic spectrum sensing functions, among others.

- **Distinguishing mobility from PU activity**: In classical ad hoc networks, routes formed over multiple hops may periodically experience disconnections caused by node mobility. These cases may be detected when the next hop node in the path does not reply to messages and the retry limit is exceeded at the link layer. However, in CRAHNs, a node may not be able to transmit immediately if it detects the presence of a PU on the spectrum, even in the absence of mobility. Thus, correctly inferring mobility conditions and initiating the appropriate recovery mechanism in CRAHNs necessitate a different approach from the classical ad hoc networks.
1.3 Spectrum management and cognitive cycle

The components of the CRAHN architecture, as shown in Figure 1.5, can be classified in two groups: the primary network and the CRN components. The primary network is referred to as an existing network, where the PUs have a license to operate in a certain spectrum band. If primary networks have an infrastructure support, the operations of the PUs are controlled through primary base stations. Due to their priority in spectrum access, the PUs should not be affected by unlicensed users. The CRN (or secondary network) does not have a license to operate in a desired band. Hence, additional functionality is required for CR users (or secondary user) to share the licensed spectrum band. Also, CR users are mobile and can communicate with each other in a multi-hop manner on both licensed and unlicensed spectrum bands. Usually, CRNs are assumed to function as stand-alone networks, which do not have direct communication channels with the primary networks. Thus, every action in CRNs depends on their local observations.

In order to adapt to dynamic spectrum environment, the CRAHN necessitates the spectrum-aware operations, which form a cognitive cycle [10]. As shown in Figure 1.6, the steps of the cognitive cycle consist of four spectrum management functions: spectrum sensing, spectrum decision, spectrum sharing, and spectrum mobility. To implement CRAHNs, each function needs to be incorporated into the classical layering protocols. The following are the main features of spectrum management functions:

- **Spectrum sensing**: A CR user can be allocated to only an unused portion of the spectrum. Therefore, a CR user should monitor the available spectrum bands, and then detect spectrum holes. Spectrum sensing is a basic functionality in CR networks, and hence it is closely related to other spectrum management functions as well as layering protocols to provide information on spectrum availability.

- **Spectrum decision**: Once the available spectrums are identified, it is essential that the CR users select the most appropriate band according to their QoS requirements. It is important to characterize the spectrum
band in terms of both radio environment and the statistical behaviors of the PUs. In order to design a decision algorithm that incorporates dynamic spectrum characteristics, we need to obtain a priori information regarding the PU activity. Furthermore, in CRAHNs, spectrum decision involves jointly undertaking spectrum selection and route formation.

- **Spectrum sharing**: Since there may be multiple CR users trying to access the spectrum, their transmissions should be coordinated to prevent collisions in overlapping portions of the spectrum. Spectrum sharing provides the capability to share the spectrum resource opportunistically with multiple CR users which includes resource allocation to avoid interference caused to the primary network. For this, game theoretical approaches have also been used to analyze the behavior of selfish CR users. Furthermore, this function necessitates a CR medium access control (MAC) protocol, which facilitates the sensing control to distribute the sensing task among the coordinating nodes as well as spectrum
access to determine the timing for transmission.

- **Spectrum mobility**: If a PU is detected in the specific portion of the spectrum in use, CR users should vacate the spectrum immediately and continue their communications in another vacant portion of the spectrum. For this, either a new spectrum must be chosen or the affected links may be circumvented entirely. Thus, spectrum mobility necessitates a spectrum handoff scheme to detect the link failure and to switch the current transmission to a new route or a new spectrum band with minimum quality degradation. This requires collaborating with spectrum sensing, neighbor discovery in a link layer, and routing protocols. Furthermore, this functionality needs a connection management scheme to sustain the performance of upper layer protocols by mitigating the influence of spectrum switching.

To overcome the drawback caused by the limited knowledge of the network, all of spectrum management functions are based on cooperative operations where CR users determine their actions based on the observed infor-
1.3.1 Spectrum sensing for cognitive radio ad hoc networks

A cognitive radio is designed to be aware of and sensitive to the changes in its surrounding, which makes spectrum sensing an important requirement for the realization of CR networks. Spectrum sensing enables CR users to exploit the unused spectrum portion adaptively to the radio environment. This capability is required in the following cases: (1) CR users find available spectrum holes over a wide frequency range for their transmission (out-of-band sensing), and (2) CR users monitor the spectrum band during the transmission and detect the presence of primary networks so as to avoid interference (in-band sensing).

As shown in Figure 1.7, the CRAHN necessitates the following functionalities for spectrum sensing:

- **PU detection**: The CR user observes and analyzes its local radio environment. Based on these location observations of itself and its neighbors, CR users determine the presence of PU transmissions, and accordingly identify the current spectrum availability.

- **Cooperation**: The observed information in each CR user is exchanged with its neighbors so as to improve sensing accuracy.

- **Sensing control**: This function enables each CR user to perform its
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Figure 1.8: Classification of spectrum sensing

sensing operations adaptively to the dynamic radio environment. In addition, it coordinates the sensing operations of the CR users and its neighbors in a distributed manner, which prevents false alarms in cooperative sensing.

In order to achieve high spectrum utilization while avoiding interference, spectrum sensing needs to provide high detection accuracy. However, due to the lack of a central network entity, CR ad hoc users perform sensing operations independently of each other, leading to an adverse influence on sensing performance.

Since CR users are generally assumed not to have any real-time interaction with the PU transmitters and receivers, they do not know the exact information of the ongoing transmissions within the primary networks. Thus, PU detection depends on the only local radio observations of CR users. Generally, PU detection techniques for CRAHNs can be classified into three groups: primary transmitter detection, primary receiver detection, and interference temperature management (see Figure 1.8).

As shown in Figure 1.9, transmitter detection is based on the detection of the weak signal from a primary transmitter through the local observations of CR users. The primary receiver detection aims at finding the PUs that are receiving data within the communication range of a CR user [11]. As depicted in Figure 1.10, the local oscillator (LO) leakage power emitted by the
radio frequency (RF) front-end of the primary receiver is usually exploited, which is typically weak. Thus, although it provides the most effective way to find spectrum holes, currently this method is only feasible in the detection of the TV receivers. Interference temperature management accounts for the cumulative RF energy from multiple transmissions, and sets a maximum cap on their aggregate level that the primary receiver could tolerate, called an interference temperature limit [12]. As long as CR users do not exceed this limit by their transmissions, they can use this spectrum band. However, the difficulty of this model lies in accurately measuring the interference temperature since CR users cannot distinguish between actual signals from the PU and noise/interference. For these reasons, most of current research on spectrum sensing in CRAHNs has mainly focused on primary transmitter detection.

\subsection*{1.3.2 Spectrum decision for cognitive radio ad hoc networks}

CRAHNs require capabilities to decide on the best spectrum band among the available bands according to the QoS requirements of the applications. This notion is called spectrum decision and constitutes a rather important
but yet unexplored topic. Spectrum decision is closely related to the channel characteristics and the operations of PUs. Spectrum decision usually consists of two steps: first, each spectrum band is characterized based on, not only local observations of CR users, but also statistical information of primary networks. Then, based on this characterization, the most appropriate spectrum band can be chosen. Generally, CRAHNs have unique characteristics in spectrum decision due to the nature of multi-hop communication. Spectrum decision needs to consider the end-to-end route consisting of multiple hops. Furthermore, available spectrum bands in CR networks differ from one hop to the other. As a result, the connectivity is spectrum-dependent, which makes it challenging to determine the best combination of the routing path and spectrum. Thus, spectrum decision in ad hoc networks should interact with routing protocols. The following are main functionalities required for spectrum decision:

- *Spectrum characterization:* Based on the observation, the CR users determine not only the characteristics of each available spectrum but also its PU activity model.

- *Spectrum selection:* The CR user finds the best spectrum band for each hop on the determined end-to-end route so as to satisfy end-to-end QoS
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Figure 1.11: Spectrum decision structure for ad hoc CRNs

requirements.

• **Reconfiguration**: The CR users reconfigure communication protocol as well as communication hardware and RF front-end according to the radio environment and user QoS requirements.

CR ad hoc users require spectrum decision at the beginning of the transmission. As depicted in Figure 1.11, through RF observation, CR users characterize the available spectrum bands by considering the received signal strength, interference, and the number of users currently residing in the spectrum, which are also used for resource allocation in classical ad hoc networks. However, unlike classical ad hoc networks, each CR user observes heterogeneous spectrum availability which is varying over time and space due to the PU activities. This changing nature of the spectrum usage is considered in the spectrum characterization. Based on this characterization, CR users determine the best available spectrum band to satisfy its QoS requirements. Furthermore, quality degradation of the current transmission can also initiate spectrum decision to maintain the quality of a current session.

Since the available spectrum holes show different characteristics, which vary over time, each spectrum hole should be characterized by considering both the time-varying radio environment and the spectrum parameters such as operating frequency and bandwidth. Hence, it is essential to define parameters that can represent a particular spectrum band as follows:
• **Interference**: From the amount of the interference at the primary receiver, the permissible power of a CR user can be derived, which is used for the estimation of the channel capacity.

• **Path loss**: The path loss is closely related to the distance and frequency. As the operating frequency increases, the path loss increases, which results in a decrease in the transmission range. If transmission power is increased to compensate for the increased path loss, interference at other users may increase.

• **Wireless link errors**: Depending on the modulation scheme and the interference level of the spectrum band, the error rate of the channel changes.

• **Link layer delay**: To address different path loss, wireless link error, and interference, different types of link layer protocols are required at different spectrum bands. This results in different link layer delays.

It is desirable to identify the spectrum bands combining all the characterization parameters described above for accurate spectrum decision. Besides, in order to describe the dynamic nature of CRNs, we need a new metric to capture the statistical behavior of primary networks, called **PU activity**. Since there is no guarantee that a spectrum band will be available during the entire communication of a CR user, the estimation of PU activity is a very crucial issue in spectrum decision. Most of CR research assumes that PU activity is modeled by exponentially distributed inter-arrivals ([13], [14], [15]). In this model, the PU traffic can be modeled as a two state birth-death process with death rate $\alpha$ and birth rate $\beta$. An ON (Busy) state represents the period used by PUs and an OFF (Idle) state represents the unused period. Since each user arrival is independent, each transition follows the Poisson arrival process. Thus, the length of ON and OFF periods are exponentially distributed. There are some efforts to model the PU activity in specific spectrum bands based on field experiments. In [16], the characteristics of primary usage in cellular networks are presented based on the call records collected by network systems, instead of real measurement. This analysis shows that an exponential call arrival model is adequate to capture
the PU activity, while the duration of wireless voice calls does not follow an exponential distribution. The above approaches are fixed models based on off-line measurements. Hence, they do not adequately capture the time varying nature of the PU activity. In addition, similar to the classical Poisson model, these approaches fail to capture the bursty and spiky characteristics of the monitored data [17]. However, as mentioned in [16], accounting for the short term fluctuations is also important so that CR users can accurately detect more transmission opportunities. In order to accurately track the changing PU activity, a novel real-time based PU activity model for CR networks is developed in [18]. Here, the PU signal samples are first collected over a pre-determined duration. Then, the observed PU signals are clustered together, if they are greater than a threshold. Based on this clustering, the current PU arrival-departure rates can be estimated. The duration of collecting the signal samples, as well as the threshold for classifying the observed value as a legitimate PU signal are calculated in this work. However, this approach needs several PU signal samples collected at one centralized location. Thus, this needs to be extended for CRAHNs, so that each CR user may form individual clusters of the PU signals, based on their local observation, which can then be combined to give the complete PU activity model. Moreover, the additive white Gaussian noise (AWGN) channel model used in the proposed approach does not incorporate the effects of fading and shadowing, which can lower the accuracy of the PU activity prediction.

1.3.3 Spectrum sharing for cognitive radio ad hoc networks

The shared nature of the wireless channel necessitates coordination of transmission attempts between CR users. In this respect, spectrum sharing provides the capability to maintain the QoS of CR users without causing interference to the PUs by coordinating the multiple access of CR users as well as allocating communication resources adaptively to the changes of radio environment. Thus, spectrum sharing is performed in the middle of a communication session and within the spectrum band, and includes many functionalities of a medium access control (MAC) protocol and resource al-
location in classical ad hoc networks. However, the unique characteristics of cognitive radios, such as the coexistence of CR users with PUs and the wide range of available spectrum, incur substantially different challenges for spectrum sharing in CRAHNs. Spectrum sharing techniques are generally focused on two types of solutions, i.e., spectrum sharing inside a CR network (intra-network spectrum sharing), and among multiple coexisting CRNs (inter-network spectrum sharing). However, since the CRAHNs do not have any infrastructure to coordinate inter-network operations, they are required to consider the only intra-network spectrum sharing functionality. Furthermore, similar to spectrum sensing, the CR users need to have all CR sharing capabilities due to the lack of a central entity. Thus, all decisions on spectrum sharing need to be made by CR users in a distributed manner. Fig. 13 depicts the functional blocks for spectrum sharing in CRAHNs. Spectrum sharing shares some functionalities with spectrum sensing in CRAHNs as follows:

- **Resource allocation**: Based on the QoS monitoring results, CR users select the proper channels (channel allocation) and adjust their transmission power (power control) so as to achieve QoS requirements as well as resource fairness. Especially, in power control, sensing results need to be considered so as not to violate the interference constraints.

- **Spectrum access**: It enables multiple CR users to share the spectrum resource by determining who will access the channel or when a user may access the channel. This is (most probably) a random access method due to the difficulty in synchronization.

Once a proper spectrum band is selected in spectrum decision, communication channels in that spectrum need to be assigned to a CR user while determining its transmission power to avoid the interference to the primary network (resource allocation). Then the CR user decides when the spectrum should be accessed to avoid collisions with other CR users (spectrum access).
1.3.4 Spectrum mobility for cognitive radio ad hoc networks

CR users are generally regarded as visitors to the spectrum. Hence, if the specific portion of the spectrum in use is required by a PU, the communication needs to be continued in another vacant portion of the spectrum. This notion is called spectrum mobility. Spectrum mobility gives rise to a new type of handoff in CR networks, the so-called spectrum handoff, in which, the users transfer their connections to an unused spectrum band. In CRAHNs, spectrum handoff occurs: (1) when PU is detected, (2) the CR user loses its connection due to the mobility of users involved in an on-going communication, or (3) with a current spectrum band cannot provide the QoS requirements. In spectrum handoff, temporary communication break is inevitable due to the process for discovering a new available spectrum band. Since available spectrums are dis-contiguous and distributed over a wide frequency range, CR users may require the reconfiguration of operation frequency in its RF front-end, which leads to significantly longer switching time. The purpose of the spectrum mobility management in CRAHNs is to ensure smooth and fast transition leading to minimum performance degradation during a spectrum handoff. Furthermore, in spectrum mobility, the protocols for different layers of the network stack should be transparent to the spectrum handoff and the associated latency, and adapt to the channel parameters of the operating frequency. Another intrinsic characteristic of spectrum mobility in CR networks is the interdependency with the routing...
protocols. Similar to the spectrum decision, the spectrum mobility needs to involve the recovery of link failure on the end-to-end route. Thus, it needs to interact with routing protocols to detect the link failure due to either user mobility or PU appearance. The main functionalities required for spectrum mobility in the CRAHN are:

- **Spectrum Handoff**: The CR user switches the spectrum band physically and reconfigures the communication parameters for an RF front-end (e.g. operating frequency, modulation type).

- **Connection management**: The CR user sustains the QoS or minimizes quality degradation during the spectrum switching by interacting with each layering protocols.

As stated previously, the spectrum mobility events can be detected as a link failure caused by user mobility as well as PU detection. Furthermore, the quality degradation of the current transmission also initiates spectrum mobility. When these spectrum mobility events are detected through spectrum sensing, neighbor discovery, and routing protocol, they trigger the spectrum mobility procedures. Figure 1.13 illustrates the functional blocks for spectrum mobility in CRAHNs. By collaborating with spectrum decision, a CR user determines a new spectrum band on the determined route, and switch its current session to the new spectrum (spectrum handoff). During the spectrum handoff, the CR user need to maintain current transmission not to be interfered by the switching latency.

### 1.4 Common control channel

The common control channel (CCC) is used for supporting the transmission coordination and spectrum related information exchange between the CR users. It facilitates neighbor discovery, helps in spectrum sensing coordination, control signaling and exchange of local measurements between the CR users. The operation of the CCC is different from the data transmission over the licensed band in the following aspects:
CR users may optimize their channel use over a number of constraints, such as channel quality, access time, observed PU activity, network load, among others during CR data transmission. However, these parameters are not known to the CR users in advance at the start of the network operation, and thus, it is a challenge to choose the CCC with the minimum or no exchange of network information.

Spectrum bands that are currently used for data transfer may suddenly become unavailable when a PU appears. While the data communication is interrupted, the affected CR users need to coordinate a new spectrum that does not interfere with the PUs on either end of the link. This control information used in the new spectrum selection must be sent reliably and thus, an always on CCC is needed.

Figure 1.14 shows the different design approaches that may be followed for establishing and using the CCC. The two main approaches are in-band and out-of-band CCC, depending on whether the control channel shares the data channel or uses a dedicated spectrum, respectively. For inband operation, the range of the CCC is limited to local coverage. As opposed to this, out-of-band CCC may have dedicated spectrum assigned as a constant through the network, i.e. global coverage, or may use different regionspecific bands, i.e. local coverage.
The licensed spectrum used for ongoing data transmission band may be used to transmit the control messages for the case of in-band signaling ([19], [20], [21]). In this case, the CCC operation is only for a specific purpose and for a temporary duration. Moreover, each node pair may use a different channel for communication. As the CCC is the same as the channel used for data, the extent of coverage of the CCC is local, i.e. unique to the corresponding node pair. The advantage of this approach is that a separate dedicated transceiver is not needed for the CCC. Moreover, there is no added spectrum switching cost in single transceiver systems, as they do not need to frequently change the spectrum for control and data messages. While the in-band CCC simplifies the coordination protocol between the CR users, there are several drawbacks to using this approach. Firstly, the CCC is affected whenever a PU reclaims the operational spectrum. At this time, the new spectrum acceptable to both ends of an active link needs to be identified and this exchange of information is difficult without an available CCC. Secondly, the control messages may affect the data transmission and reduce the end-to-end throughput. Moreover, as the channel used for CCC changes frequently, and hence new CR users that join the network may have a considerable initial setup time to find the channel for sending their respective join requests. Out-of-band signaling (through a licensed channel reserved for CCC use or by using the unlicensed band) minimizes the CCC disruptions caused by PU activity. In this case, the spectrum reservation for the CCC may either be made for a short duration, or there may be a permanent assignment. As an example, the spectrum sensing function may necessitate quiet periods in the neighborhood of the sensing node or integrating measured values from several different sources, so that the transmitted power of the PUs may be accurately detected. At such times, the CCC may be set up for coordinating these quiet periods with the other CR users, and communicating the sensed information back to the initiating node. After the sensing procedure is complete, the CCC is no longer active and the spectrum can be reclaimed for data transmission. As the data and the control signaling are separate, more than one transceiver may be needed for dedicated CCC monitoring. For single radio devices, the cost of switching between the data band and the CCC, and the associated deaf period when the CCC is not sensed, must be
CHAPTER 1. COGNITIVE RADIO NETWORKS

Figure 1.14: Common control channel design classification

accounted for in the protocol design. The different types of out-of-band CCC design approaches are mentioned below:

- **Local coverage**: CR users may be grouped into clusters and a CCC may be used for all the nodes in the same cluster. This grouping of nodes may be based on their physical proximity, spectrum usage conditions, and other common environmental factors ([22], [23]). In the ideal case, the set of nodes using the CCC should be varied (and hence, the number of active CCCs in the network) to reflect the changing spectrum conditions, with the best case being a network wide common CCC.

- **Global coverage**: For the CCCs that have global coverage ([24], [25], [26]), the channel for communication must be carefully chosen so that they are not interrupted over long periods of time. While this considerably simplifies the CCC operation, there are some drawbacks of this method. The PU activity varies from one geographical region to another, and hence, it is difficult to identify a CCC that is global, or uniformly acceptable throughout the entire network. In addition, collecting and disseminating this information to all the CR users in a distributed manner involves repeated networkwide flooding.
Bibliography


Chapter 2

Related work

2.1 Handled problems

In the previous chapter I described the main features of a CRAHN, analyzing the functions that a cognitive node has to accomplish to take advantages of the dynamic spectrum access. In this chapter, instead, I focus the attention on two aspects that need to be revised in a cognitive scenario: connectivity and routing. The key distinguish factors of the secondary networks is that the topology is highly dynamic and nodes should cooperate in a distributed fashion to achieve network connectivity and communication. The fact that the communication links of the secondary networks vary as a function of the PUs activities makes the design and the analysis of these networks very challenging. In classical wireless Ad-Hoc network nodes communicate by using the same frequency and distance among nodes combined with the adopted transmission power are the main parameters affecting network connectivity. On the contrary, in a CRAHN also the physical location of and the activity the PUs affect the SU network topology. In fact, in a CRAHN two SUs can connect if they are in radio visibility and have at least one available common spectrum hole. As a consequence, not only the nodes position and transmission power but also their free spectrum holes affect network connectivity. Consequently the concept of connectivity has to be revised and worked up, in order to take into account this aspect. A simple example is depicted in Figure 2.1 representing three PUs ($PU_1$, $PU_2$ and $PU_3$), each operating on a
different portion of the spectrum and having a different coverage area ($CA_3$). Six SUs share the spectrum with the PUs. They can potentially transmit on all the three frequency bands but two nodes can communicate if they have at least one available common channel $c$. In Figure 2.1(a), only two PUs are active (1 and 2), the available common channels are reported beside each node and the resulting topology is shown. Nodes in $CA_1$ are inhibited from using the channel $c_1$ while nodes in $CA_2$ cannot access the channel $c_2$. The consequence is that all six nodes share a subset of the three spectrum bands and the resulting physical topology, on the basis also of the maximum transmission range of each node, is shown in the Figure 2.1(a). Figure 2.1(b) shows the physical topology after the activation of $PU_3$ that inhibits the use of channel $c_3$ for nodes in $CA_3$. In this case, nodes in the coverage area $CA_3$ (i.e., nodes F, E and D) can no longer use the spectrum band $c_3$. Consequently, node F and D cannot communicate any more. Moreover, node B misses the opportunity to communicate with F and D, as well, since the only band available for this communication was the band $c_3$. This simple example shows that the presence of PUs impacts on secondary network topology, and consequently its connectivity.

Moreover, the dynamic variation of the topology and connectivity has a significant impact on communication reliability as well as on the design of

Figure 2.1: Network topology affected by PUs activity: (a) the PU in the coverage area $CA_3$ is not active and (b) the PU in the coverage area $CA_3$ is active.
routing schemes: routing has to cope with spectrum management to exploit spectrum holes and to guarantee a not-disconnected communication, caused by the dynamic spectrum heterogeneity perceived by SUs. In other words, the problem of routing in multi-hop CRNs targets the creation and the maintenance of wireless multi-hop paths among SUs by deciding both the relay nodes and the spectrum to be used on each link of the path. Such problem exhibits similarities with routing in multi-channel, multi-hop ad hoc networks and mesh networks, but with the additional challenge of having to deal with the simultaneous transmissions of the PUs which dynamically change the spectrum opportunities availability. In this thesis I consider the implications of the spectrum heterogeneity on the CRAHN connectivity and on the routing. To this aim I leverage the graph theory by studying the Laplacian spectrum of the CRAHN graph where the PUs dynamics are considered. I introduce the cognitive algebraic connectivity, i.e., the second smallest eigenvalue of the Laplacian of a graph, in a cognitive scenario. With this model I contribute to:

1. provide a methodology for evaluating the CRAHN connectivity (Chapter 3);

2. build up an utility function which is shown to be effective for capturing some key characteristics of networks paths and can be used to compare them for routing purposes (Chapter 4).

The main advantage of this model is that I am able to have a unique metric, that is the algebraic connectivity, which captures the network connectivity, the average distance of nodes, the network diameter. Besides the connectivity of the whole network, this metric can be used also to evaluate routes. In fact, by elaborating the algebraic connectivity in a cognitive scenario, this metric is shown to be able to give a lower weight to routes where nodes are continuously interrupted by PUs transmissions; these paths become longer with respect to paths where nodes have more spectrum availability and are not selected during the traditional routing decision, that is normally based on hop count criteria. I then design a routing scheme which captures the connectivity characteristics of the paths and suitably selects the best route in uncertain and high variable scenarios.
2.2 Related work on connectivity

While the connectivity of homogeneous ad hoc networks consisting of peer users has been well studied (see, for example, [1], [2], [3], [4], [5], [6], [7]), little is known about the connectivity of heterogeneous networks. The problem is fundamentally different from its counterpart in homogeneous networks. In particular, the connectivity of the low-priority network component depends on the characteristics (traffic pattern/load, topology, interference tolerance, etc.) of the high-priority component, thus creating a much more diverse and complex design space. There are a number of classic results on the connectivity of homogeneous ad hoc networks. For example, it has been shown that to ensure either 1-connectivity (there exists a path between any pair of nodes) ([3], [4]) or k-connectivity (there exist at least k node-disjoint paths between any pair of nodes) [6], the average number of neighbors of each node must increase with the network size. On the other hand, to maintain a weaker connectivity p-connectivity (i.e., the probability that any pair of nodes is connected is at least p), the average number of neighbors is only required to be above a certain magic number which does not depend on the network size [5]. The theory of continuum percolation has been used by Dousse et al. in analyzing the connectivity of a homogeneous ad hoc network under the worst case mutual interference ([1], [2]). In [7], the connectivity and the transmission delay in a homogeneous ad hoc network with statically or dynamically on-off links are investigated from a percolation-based perspective. The optimal power control in heterogeneous networks has been studied in [8], which focuses on a single pair of SUs in a Poisson network of PUs. The impacts of SUs transmission power on the occurrence of spectrum opportunities and the reliability of opportunity detection are analytically characterized. A recent works in [9], [10] and [11] addressed the connectivity of large-scale CRAHNs where SUs exploit channels unused by PUs. In this work authors, by using techniques and theories in continuum percolation, characterize the connectivity region where the secondary network is connected and discuss the tradeoff between proximity (the number of neighbors) and the occurrence of spectrum opportunities. The result of the work is the specification of the profile of the connectivity region as a function of the densities of the SUs.
and of the primary transmitters. Besides, an analysis of the impact of the transmission power is provided. These results have a key importance for the CRAHNs dimensioning. In [12] authors consider channel randomness and interference jointly in their connectivity study. In particular, they compute the node isolation probability, an important connectivity metric in ad hoc networks and demonstrate the tradeoff between the capacity and connectivity in connection with the transmitter density. However the study of node isolation probability is different from the study of the probability of connected networks.

The major drawback of these papers is in the network model, since they assume that a SU can not use the licensed spectrum band of a PU when the SU belongs to the interference area of the PU: in this way the SU misses opportunity to transmit, since it could use the licensed band of that PU when this is inactive. In my network model, presented in the Chapter 3, I tried to improve this aspect, in order to exploit not only the spatial diversity, but also the temporal one.

2.3 Related work on routing

Several recent works have analyzed and proposed routing algorithms for cognitive radio ad hoc networks [13]. The routing solutions can be classified in two main categories:

1. approaches based on static network topologies, with fully available topological information on SUs and PUs;

2. approaches based on partial and local information about the network state, both in terms of PUs presence and activity.

In the former case, a spectrum occupancy map is available to the network nodes, or to a central control entity, which could be represented by the centrally-maintained spectrum data bases recently promoted by the FCC to indicate over time and space the channel availabilities [10] in the spectrum below 900 MHz and around 3 GHz. The considered architectural model is a static cognitive multi-hop network where the spectrum availability between
any given node pair is known. The routing approaches building on this assumption leverage theoretical tools to design efficient routes, differentiating on the basis of which kind of theoretical tool is used to steer the route design. A first class encompasses all solutions based on a graph abstraction of the cognitive radio network. The second sub-class instead employs mathematical programming tools to model and design flows along the cognitive multi-hop network. Although these approaches are often based on a centralized computation of the routing paths, their relevance is in the fact that they provide upper bounds and benchmarks for the routing performance. On the other hand, routing schemes based on local spectrum knowledge include all those solutions where information on spectrum availability is locally constructed at each SU through distributed protocols. Thus, the routing module is tightly coupled to the spectrum management functionalities. Indeed, besides the computation of the routing paths, the routing module should be able to acquire network state information, such as currently available frequencies for communication and other locally available data, and exchange them with the other network nodes. While the network state in traditional ad hoc networks is primarily a function of node mobility and traffic carried in the network, network state in multihop CRNs is also influenced by PU activity. How this activity is and which are the suitable models to represent it are key components for the routing design.

In case 1), routing solutions have been combined with spectrum allocation schemes mainly based on graph theory [14],[15],[16] and mixed-integer linear/non linear programming frameworks [17]. The implementation of these approaches may result complex and may present a low scalability, however their importance can be seen in the application to all those scenarios where the SUs have access to databases storing the spectrum maps, as recently envisaged by the FCC for the use of TV bands [10]. Similarly to these works, in this thesis I leverage the graph theory by proposing an approach for evaluating the connectivity of the cognitive radio. This mathematical framework is based on the Laplacian of the graph modeling the network and it is used with a twofold aim: on one side we provide a tool for evaluating the connectivity of a cognitive radio network (or of a part of it) that can be used in semi-static scenarios where the PUs behavior and location are known. On the other side
Figure 2.2: Classification of cognitive routing schemes as described in [13].

I use this approach for designing a routing metric able to capture the connectivity of cognitive radio routes where the PUs dynamics are considered. In both cases I propose a methodology for reducing the computational complexity for computing the average value of the network connectivity which is related to the statistical behavior of the PUs.

In the second routing category instead there exist several approaches based on local information on spectrum occupancy gathered by each SU through local and distributed sensing mechanisms. A further classification of the proposals in the local spectrum knowledge family can be based on the specific measure of the route quality used to set up quality routes. Four classes can be broadly recognized: from left to right in Figure 2.2, we have routing solutions aiming at controlling the interference the multi-hop CRNs create, delay-based and throughput-based routing schemes where the routing module targets the minimization of the end-to-end delay and the maximization of the achievable throughput, respectively; and finally, those solutions where the quality of the paths is strictly coupled to its availability over time and to its stability.

A distinctive characteristic of all routing approaches belonging to case 2), is that they combine to the routing the selection of the spectrum on each link of the path. This can be done by using different metrics for capturing the
CHAPTER 2. RELATED WORK

characteristics of the available spectrum holes. In this latter category specific emphasis is dedicated to the inclusion of spectrum availability in the routing metric. The Spectrum Aware Mesh Routing (SAMER) [18] is a protocol that accounts for long term and short term spectral availability. SAMER seeks to utilize available spectrum blocks by routing data traffic over paths with higher spectrum availability, without ignoring instantaneous spectral conditions. To this aim authors define a metric for estimating Path Spectrum Availability (PSA) and utilize this metric to route data traffic over paths with higher spectrum availability. The paper of Ding et al. [19] proposes an algorithm searching for the appropriate spectrum bands with the aim of maximizing the throughput jointly considering the system power constraints. In the proposed approach, named ROSA (ROuring and Spectrum Allocation algorithm), each cognitive device makes real-time decision on spectrum and power allocation based on locally collected information. The building blocks of ROSA are: i) the discovery of feasible next hops; ii) the selection of the hop that maximizes a suitable utility function; iii) the power selection for accessing the medium on the selected hop. The combination of these three blocks gives rise to a cross-layered routing solution.

Finally, also the behaviors of the PUs is a key parameter to be considered for routing data in a cognitive radio. In fact, routes must explicitly provide a measure of protection to the ongoing communication of the PUs, while at the SUs side must guarantee stability when the PU behavior varies. This is taken into account in a set of routing solutions where the PUs’ statistical behavior and the consequent spectrum fluctuations are considered via suitable models in the routing metrics [20] [21]. Besides this, also the ability to reconfigure the routing paths when a PU becomes active can be a distinctive feature of the routing. A work with this ability is the one presented in [22] where the routing scheme is based on the cost of maintaining a connection as a sequence of paths that may vary their availability to the PUs activities. We also consider spectrum availability in the route set up procedure, but, differently than [22], rather than reacting to changes in the spectrum availability, our approach proactively captures route stability in the routing metric, thus minimizing the route reconfiguration a priori.
Bibliography


Chapter 3

Cognitive algebraic connectivity

3.1 Basic scenario description

I consider a so called underlay cognitive radio network where $S$ SUs coexist with $P$ PUs. The secondary network is assumed to work in a multi-hop fashion: the secondary traffic is transferred via multi-hop routes.

As for the primary network, I assume that each $PU_p$ ($p = 1, \ldots, P$) has a licensed access to a given Spectrum Band (SB), denoted as $SB_p$, also named channel in the rest of the thesis. Without loss of generality I assume that the number of different spectrum bands is equal to $P$ and the spectrum band $SB_p$ is only licensed for $PU_p$. A PU is constituted by a Primary Transmitter (PT) and a set of Primary Receivers (PRs). The PT can model the base station of a GSM or UMTS network as well as the antenna of a TV broadcaster. The PRs are the end terminals of the PU network. I model each $PU_p$ with the relative Coverage Area ($CA_p$) which represents the area where a licensed transmitter or receiver is present. I consider the $CA$ circularly shaped centered in the PT and with radius $r_{PU}$, that represents the transmission range of the PT. I associate to each $PU_p$ a binary aleatory variable $b_p$ that represents the activity state of $PU_p$, that is:

$$b_p = \begin{cases} 
1 & \text{if the } PU_p \text{ is active} \\
0 & \text{otherwise} 
\end{cases} \quad (3.1)$$
Consequently, I can characterize each PU$_p$ with its average activity factor:

$$a_p = E[b_p] \quad (3.2)$$

The average activity factor of each PU can be numerically calculated by measuring the average duration of the PU activity period and the average duration of the relative silence period. The probability that PU$_p$ is inactive on its licensed spectrum band is $1 - a_p$. In the literature there exist some studies on the average utilization of the different spectrum bands. One very interesting work is the paper of Wellens and Mähönen [2] where measures and models on the spectrum occupancy in different spectrum bands are provided. This study shows that busy or completely vacant spectrum bands are very probable and provides some distributions to model the spectrum occupancy in different radio environments like GSM900, GSM1800, UMTS, TV, ISM at 2.4 GHz.

As for the secondary network, I assume that each SU$_s$ ($s = 1, \ldots, S$) transmits with a given transmission range $r_{SU_s}$. To reflect the characteristic of the real wireless communication environment, I use for the secondary network the Log-Normal Shadowing Model (LNSM) [3], where the signal strength perceived by a certain node not only depends on the distance between transmitter and receiver, but also includes some random factors, such as the presence of obstacles. In this kind of model, nodes that are located within the transmission range of the sender node can receive a packet with high probability, instead nodes outside the transmission range of the sender node can also receive the packet, but with low probability. More precisely, I assume that each SU can receive a packet sent from another node SU$_s$ having transmission range $r_{SU_s}$, that is located $d$ meters away, with probability $P(d)$, defined as follows:

$$P(d) = \begin{cases} 
1 - \frac{(d/r_{SU_s})^{2\alpha}}{2} & \text{if } 0 < d \leq r_{SU_s} \\
\frac{(2r_{SU_s} - d)^{2\alpha}}{2} & \text{if } r_{SU_s} < d \leq 2 \cdot r_{SU_s} \\
0 & \text{otherwise}
\end{cases} \quad (3.3)$$
where \( 2 \leq \alpha \leq 4 \) is the path loss exponent. In Figure 3.1 it is shown an example of the receiving probability \( P(d) \) with the LNSM supposing that \( r_{SU} = 100 \) m and \( \alpha = 2 \).

I assume that:

- two SUs (SU \(_i\) and SU \(_j\)) can communicate if their Euclidean distance is lower than the minimum between their transmission ranges \( r_{SU_i} \) and \( r_{SU_j} \), that is if the receiving probability at both side is higher than 0.5;

- the transmission of a SU \(_i\) influences the reception of a PR if their Euclidean distance is lower than \( 2 \cdot r_{SU_i} \), that is if receiving probability is higher than 0.

On the basis of these assumptions, I characterize each SU \(_s\) with a Transmission Area \( TA_s \), that is the area circularly shaped centered in the SU \(_s\) and with radius \( r_{SU_s} \), and with an Influence Area \( IA_s \), that is the area circularly shaped centered in the SU \(_s\) and with radius \( 2 \cdot r_{SU_s} \) (see Figure 3.2). Obviously I have that \( TA_s \subset IA_s \).

I assume that SUs are equipped with a cognitive radio device that can be tuned to the different spectrum bands in order to sense them, verify the availability and transmit/receive on them. I assume an ideal spectrum sensing able to measure the PUs activities on the different spectrum bands [1]. I also suppose that SUs can potentially use all the \( SB_p \) spectrum bands, with \( p = 1, ..., P \). In fact, each SU opportunistically exploits locally unused licensed spectrum bands without interfering with PUs. In order to deduce
when a spectrum band $SB_p$ is available for SU$s$, I better specify when the transmission of SU$s$ influences PU$p$. I assume that this happens when SU$s$’s Influence Area overlaps, even partially, the PU$p$’s Coverage Area. This means that PU$p$ is influenced not only by those SUs belonging to $CA_p$, but also by those SUs not in $CA_p$ but having their IAs intersecting $CA_p$. This latter case to represent the fact that a transmission of these SUs can influence a PR even when the SUs are outside the CA. Indeed interference of SUs to PUs in our model is represented by the IA.

Therefore, I can state that the spectrum band $SB_p$ is available for SU$s$ when:

- SU$s$ does not influence PU$p$;
- SU$s$ could influence PU$p$, but PU$p$ is inactive.

Figure 3.3 sketches the considered reference scenario where PUs’ CA and SUs’ IA are reported. In this example two SUs (SU$_1$ and SU$_2$) are in the $CA_1$, so they cannot use $SB_1$ if PT$_1$ is active. Also SU$_3$ and SU$_4$ cannot use the spectrum band $SB_1$ if PT$_1$ is active since $IA_3$ and $IA_4$ overlap $CA_1$ and consequently SU$_3$ and SU$_4$’s transmission can interfere with one PR in $CA_1$. On the contrary, SU$_5$ can use $SB_1$, while it cannot use $SB_2$ since its transmission has an influence on $CA_2$. 


3.1.1 Discussion on the used interference model

I observe that I characterize the interference relationship among SUs and PUs by using the so-called protocol model, also known as Unit Disk Graph model (UDG) [4]. This model has been widely used as a way to simplify the mathematical characterization of physical layer. Under the protocol model, a successful transmission occurs when a node falls inside the transmission range of its intended transmitter and fall outside the interference ranges of other non-intended transmitters. That is, if a node falls in the interference range of a non-intended transmitter, then this node is considered to be interfered and thus cannot receive correctly from its intended transmitter; otherwise, the interference is assumed to be negligible.

Another widely used interference model is the physical model, also known as the SINR model ([5]), that is based on practical transceiver designs of communication systems and treats interference as noise. Under the physical model, a transmission is successful if and only if signal-to-interference-and-noise-ratio (SINR) at the intended receiver exceeds a threshold $\beta$ so that the transmitted signal can be decoded with an acceptable bit error probability. In wireless communication this model is considered as a reference model for physical layer behavior, since it realistically captures physical interference constraints, but its application in wireless network is limited by its complex-
I observe that in the proposed framework the choice of the interference model impacts on channels’ availability perceived by SUs. In fact, I assume that SU\textsubscript{s} can always use channel SB\textsubscript{p} if it does not interfere with PU\textsubscript{p}, whereas, if SU\textsubscript{s} interferes with PU\textsubscript{p}, then it can use the channel SB\textsubscript{p} only when PU\textsubscript{p} is inactive. By using the protocol model there is a binary decision of whether or not SU\textsubscript{s} interferes with PU\textsubscript{p}. Hence, with respect to the physical model, there could be two cases when the interference is not revealed in an accurate manner:

1. when SU\textsubscript{s}’s IA overlaps, even partially, the PU\textsubscript{p}’s CA, I assumes that there could be some PRs\textsubscript{p} that are not correctly receiving from PT\textsubscript{p}, due to interference generated by the transmission of SU\textsubscript{s}, and consequently the channel SB\textsubscript{p} is available for SU\textsubscript{s} only when PU\textsubscript{p} is inactive: however, this is overly conservative, since the SINR perceived by PRs\textsubscript{p} could still be over the threshold $\beta$;

2. when SU\textsubscript{s}’s IA does not overlap the PU\textsubscript{p}’s CA, I assume that SU\textsubscript{s} does not interfere with PU\textsubscript{p}, and consequently the channel SB\textsubscript{p} is always perceived available by SU\textsubscript{s}: this is somewhat optimistic, since small interference from different transmitters can aggregate and may not be negligible for some PRs\textsubscript{p}.

As for case 1), it could happen that SU\textsubscript{s} could always use the channel SB\textsubscript{p} without interfering with PRs\textsubscript{p}. Therefore, if this situation happens, I lose transmission opportunities for SU\textsubscript{s}, but it is preferable to be conservative rather than to disturb PU\textsubscript{p}.

Instead, as for case 2), from a physical point of view, it could happen that SU\textsubscript{s} disturbs some PRs\textsubscript{p} even if its IA does not overlap with PU\textsubscript{p}’s CA. Hence, in this situation, I risk to give a transmission opportunity to SU\textsubscript{s}, generating an interference on PU\textsubscript{p}, that is not allowed by the cognitive paradigm. In order to mitigate this effect, I modeled the transmission of a SU by using the LNSM and designed the SU’s IA on the basis of this model. Besides, in literature there are some works that tries to reconcile the tension between physical model and protocol model. In particular, in [6] authors investigate how to correctly use the protocol interference model in multi-hop wireless
CHAPTER 3. COGNITIVE ALGEBRAIC CONNECTIVITY

networks. They show that by combining the so-called reality check mechanism and appropriate setting of the interference range in the protocol model, it is possible to obtain comparable results under both models. So, it is also possible to apply this solution in order to reduce the negative effect on channels’ availability perceived by SUs that can arise in case 2).

Even if the protocol model not always fits with the reality of wireless networks, in the proposed framework it allows to capture both the spectrum heterogeneity and the dynamism of the PUs’ activities that are fundamental for the connectivity of the secondary network. Besides, the low computational complexity of this model counterbalances the loss of accuracy with respect to the physical model.

Moreover, in cognitive literature, more simplistic models are assumed with respect to this UDG. In fact in that models SUs are inhibited from transmission when they are in the PU’s CA. Instead with the proposed model I better catch the cognitive paradigm by leveraging transmission opportunities also in the CA.

3.2 Mathematical framework for CRAHN connectivity

In graph theory the connectivity of a network is evaluated by using the Laplacian matrix. In this Section I provide some definitions and theorems related to the connectivity analysis in graph theory and I introduce a model used for evaluating the connectivity of a CRAHN.

3.2.1 Laplacian matrix and connectivity in graph theory

Let $G(N, E)$ be a simple graph, where $N$ is the set of nodes and $E$ is the set of edges. This graph is characterized by two matrices:

- *Adjacency matrix*, denoted as $A$, that is a $N \times N$ binary matrix where
The adjacency matrix of simple bidirectional graph is symmetric and has all diagonal elements equal to 0;

- **Degree matrix**, denoted as $\mathbb{D}$, that is a $N \times N$ diagonal matrix. The $i^{th}$ element of the diagonal is the degree of node $i$ in graph $G$, denoted by $\text{deg}_i$, and it is equal to the number of edges incident on $i$; i.e., $\text{deg}_i = \sum_{k=1}^{N} a_{kj} = \sum_{k=1}^{N} a_{jk}$.

From these two matrices it is possible to derive the **Laplacian matrix** $\mathbb{L}$ of a graph $G$. It is a $N \times N$ matrix where the generic element $l_{ij}$ is:

$$l_{ij} = \begin{cases} \text{deg}_i & \text{if } j = i \\ -a_{ij} & \text{otherwise} \end{cases} \quad (3.5)$$

For a simple bidirectional graphs, $\mathbb{L}$ is symmetric, semipositive-definite and all its rows and columns sum up to 0 (that is $\mathbb{L}$ is singular).

The **Laplacian eigenvalues** of $G$ are the roots of the characteristic polynomial of $\mathbb{L}$: $p(\lambda) = \det(\mathbb{L} - \lambda \cdot \mathbb{I})$, where $\mathbb{I}$ is the identity matrix. Since $\mathbb{L}$ is symmetric and semipositive-definite, all its eigenvalues are real and non-negative. The **eigenspectrum** of $\mathbb{L}$ is the set of its eigenvalues that can be ordered from the smallest to the greatest ($\lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_N$). The notation $\lambda_k$ indicates the $k^{th}$ eigenvalue.

**Theorem 1**: The smallest eigenvalue of the Laplacian of a bidirectional graph $G$ is equal to 0 (i.e., $\lambda_1 = 0$) and the multiplicity of zeros eigenvalues is equal to the number of connected components of $G$ [7]. Consequently, $\lambda_2 = 0$ iff $G$ is disconnected: $\lambda_2$ is generally called **algebraic connectivity**.

The literature presents several properties of $\lambda_2$ related to a network modeled by a graph $G$:

- Jamakovic et al. [8] have shown that the algebraic connectivity measures stability and robustness of complex network models;
• the work of Mohar [7] discussed the relationship between the average of all distances between distinct vertices of a graph $G$ and $\lambda_2$; this average distance is inversely proportional to the algebraic connectivity.

Some considerations related to the algebraic connectivity regards the node connectivity and the edge connectivity of a graph $G$. The node connectivity of a graph $G$, denoted by $\kappa_n(G)$, is equal to the minimum number of nodes whose deletion from $G$ causes the graph to be disconnected or reduces it to a 1-node graph. A graph $G$ is $K$-node connected if $\kappa_n(G) \geq K$. Instead, the edge connectivity of a graph $G$, denoted by $\kappa_e(G)$, is equal to the minimum number of edges whose deletion from $G$ causes the graph to be disconnected or reduces it to a 1-node graph. A graph $G$ is $K$-edge connected if $\kappa_e(G) \geq K$.

Theorem 2: for any bidirectional graph $G$, the second eigenvalue of its laplacian is upper bounded by its node connectivity, which in turn is upper bounded by its edge connectivity.

$$\lambda_2 \leq \kappa_n(G) \leq \kappa_e(G)$$  \hspace{1cm} (3.6)

### 3.2.2 Expected value of algebraic connectivity in cognitive scenario

A cognitive radio ad-hoc network can be modeled by a graph. The edges of this graph vary each time a PU activates or deactivates. As a consequence the algebraic connectivity dynamically varies as a function of PUs’ behavior.

To model the secondary network, I introduce the concept of cognitive graph $G^c(S, E^c)$, where $S$ is the set of secondary nodes with cardinality $S = |S|$ and $E^c$ is the dynamic set of edges: an edge $(i, j) \in E^c$ iff nodes $i$ and $j$ are in each other’s transmission range and they have contemporarily available at least one spectrum band. By considering the variability in SUs’ spectrum band availability, the cognitive graph reflects the dynamic changes of the secondary network topology due to PUs behavior. By fixing positions and transmission ranges of SUs, the presence of an edge $(i, j)$ in the graph $G^c$ is function of the $P$ binary aleatory variables $b_p$. The number of all the possible combinations of these $P$ binary aleatory variables is $M = 2^P$.

To model if an edge $(i, j)$ is present in the graph $G^c$, I define the $P \times S$
Influence matrix $I$. This latter matrix indicates, for each PU$_p$, which are the SUs that influence it: i.e., these SUs can not use the spectrum band $SB_p$ when PU$_p$ is active. The generic element $\mu_{p,s}$ of matrix $I$ is:

$$\mu_{p,s} = \begin{cases} 1 & \text{if } \text{dist}(PU_p, SU_s) < r_{PU_p} + 2 \cdot r_{SU_s} \\ 0 & \text{otherwise} \end{cases}$$

where $\text{dist}(.)$ is the Euclidean distance.

For the $m^{th}$ combination ($m = 1, 2, ..., M$), the graph $G^c_m$, characterized by the set $E^c_m$, is obtained and consequently a cognitive Adjacency matrix $\mathcal{A}^c_m$ and a cognitive Degree matrix $\mathcal{D}^c_m$ can be provided. These matrixes vary from a combination to another. The generic element of the cognitive Adjacency matrix related to the $m^{th}$ combination $\mathcal{A}^c_m$ is:

$$a^c_{ij,m} = \begin{cases} 0 & \text{if } \text{dist}(i, j) > \min\{r_{SU_i}, r_{SU_j}\} \\ 1 & \text{if } (\text{dist}(i, j) \leq \min\{r_{SU_i}, r_{SU_j}\}) \land \\ (-\bigwedge_{p=1}^P b_{p,m} \text{ otherwise}) \end{cases}$$

The generic element of $\mathcal{D}^c_m$ is:

$$d^c_{ij,m} = \begin{cases} \sum_{k=1}^S a^c_{ik,m} & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$$

with $m = 1, 2, ..., M$.

The cognitive Laplacian matrix related to the $m^{th}$ combination $\mathcal{L}^c_m$ is obtained as difference between $\mathcal{D}^c_m$ and $\mathcal{A}^c_m$, therefore its generic element is:

$$l^c_{ij,m} = \begin{cases} d^c_{ij,m} & \text{if } i = j \\ -a^c_{ij,m} & \text{otherwise} \end{cases}$$

I indicate the second smallest eigenvalue of $\mathcal{L}^c_m$ as $\lambda^c_{2,m}$. This parameter measures the algebraic connectivity of the secondary network when the sets of active and inactive PUs are fixed and determined by the $m^{th}$ combination.
I am interested in the expected value of the second smallest eigenvalue, in the following indicated with $E[\lambda_2^c]$, since in this way I obtain a connectivity measure averaged over the random activities of PUs. $E[\lambda_2^c]$ is given by:

$$E[\lambda_2^c] = \sum_{m=1}^{M} P_m \cdot \lambda_{2,m}^c$$  \hspace{1cm} (3.11)$$

where $P_m$ is the probability of occurrence of the $m^{th}$ combination. This probability depends on the value assumed by the $P$ binary aleatory variables $b_p$ in the $m^{th}$ combination and on the different PUs’ activity factors, $a_p$. I assume that the $P$ binary aleatory variables $b_p$ are statistically independent (i.e. the activations of PU$_i$ and PU$_j$ are independent for $i, j = 1, ..., P, i \neq j$): this means that the probability of the $m^{th}$ combination is equal to the product of the probabilities that the generic binary aleatory variable $b_p$ in the $m^{th}$ combination assumes the value $b_{p,m}$.

$$P_m = \prod_{p=1}^{P} Pr(b_p = b_{p,m})$$  \hspace{1cm} (3.12)$$
where \( m = 1, 2, ..., M \). Since:

\[
P_r(b_p = b_{p,m}) = a_{b_{p,m}}^p \cdot (1 - a_p)^{1-b_{p,m}}
\]

hence

\[
P_m = \prod_{p=1}^{P} a_{b_{p,m}}^p \cdot (1 - a_p)^{1-b_{p,m}}
\]

where \( m = 1, 2, ..., M \).

With this approach it is possible to calculate the expected value of the algebraic connectivity of a secondary network with a given number of SUs and a given number of PUs characterized by their activity factors. Since the computation of \( E[\lambda_2^c] \) requires the computation of \( 2^P \) cognitive Laplacian matrices and for each of them the relative second smallest eigenvalue, in the next section I introduce a methodology that reduces the computational complexity of this average measure and obtains a good estimation of the expected value of cognitive algebraic connectivity (see Section 3.3 for a discussion on the approximation versus computational complexity). I indicate this parameter with \( \lambda_2\{E[L^c]\} \) to underline that it is obtained from only one averaged cognitive Laplacian matrix.

### 3.2.3 From \( E[\lambda_2^c] \) to \( \lambda_2\{E[L^c]\} \)

As stated before the secondary network topology changes when a PU becomes active or inactive: as a consequence it is not possible to define the set \( E^c \) in a deterministic fashion. However, I can characterize this set in a probabilistic way, indicating the probability that a generic link \((i, j)\) belongs to \( E^c \), as a function of PUs activity factors. To this aim I calculate the expected value of the \( S \times S \) cognitive Adjacency matrix \( E[A^c] \) and the expected value of the \( S \times S \) cognitive Degree matrix \( E[D^c] \), whose elements are probabilistic ones depending on PUs’ activity. In order to obtain these two matrices, I use the \( P \times S \) Influence matrix \( I \) defined in Section 3.2.2. The generic element \( E[a_{ij}^c] \) of the matrix \( E[A^c] \) represents the probability that SU\(_i\) and SU\(_j\) are connected, that is equal to:

- 0 if SU\(_i\) and SU\(_j\) are not in each other’s transmission range;
• 1 if SU\textsubscript{i} and SU\textsubscript{j} are in each other’s transmission range and they have at least one available common spectrum band regardless of PUs’ activity (this is true when there is at least one PU which is not influenced by SU\textsubscript{i} and SU\textsubscript{j});

• \( \pi \) if SU\textsubscript{i} and SU\textsubscript{j} are in each other’s transmission range but the possibility to have at least one available common spectrum band depends on PUs’ activity (0 < \( \pi \) < 1).

The term \( \pi = 1 - \prod_{p=1}^{P} a_p \) is the probability that at least one PU is inactive. This means that it is sufficient that at least one PU is inactive to guarantee the connectivity between two SUs operating in each other TA. This is due to the fact that, in our model, each SU can potentially use all the SBs and the spectrum band SB\textsubscript{p} is only licensed for PU\textsubscript{p}. Consequently, independently from which PU becomes inactive, it is sure that SU\textsubscript{i} and SU\textsubscript{j} can use the channel of that inactive PU if they are in radio visibility. In fact, for each couple of SUs in radio visibility, SU\textsubscript{i} and SU\textsubscript{j}, one of these three cases can happen:

1. PU\textsubscript{p} that becomes inactive is influenced only by SU\textsubscript{i} and not by SU\textsubscript{j}; in this case SU\textsubscript{i} and SU\textsubscript{j} can communicate by using SB\textsubscript{p}, since it is always available for SU\textsubscript{j} and in that moment it is experienced free also by SU\textsubscript{i} (see Figure 3.4);

2. symmetric case of 1);

3. PU\textsubscript{p} that becomes inactive is influenced by both SU\textsubscript{i} and SU\textsubscript{j}; in this case SU\textsubscript{i} and SU\textsubscript{j} can communicate by using SB\textsubscript{p} because this spectrum band is temporarily available for both these SUs (see Figure 3.5).

Therefore, the generic element \( E[a_{ij}^c] \) is:

\[
E[a_{ij}^c] = \begin{cases} 
0 & \text{if } \text{dist}(i,j) > \min\{r_{SU_i}, r_{SU_j}\} \\
1 & \text{if } \left(\text{dist}(i,j) \leq \min\{r_{SU_i}, r_{SU_j}\}\right) \land \\
& \left(\exists \text{ PU}_z \mid \mu_{z,i} \lor \mu_{z,j} = 0\right) \\
\pi & \text{otherwise}
\end{cases}
\]
The generic element $E[d_{ii}]$ on the main diagonal of $E[D^c]$ represents the expected value of the number of edges incident to $i$, that is the number of edges incident to $i$ weighted with the probability that each edge incident to $i$ can be actually established. As formally expressed in Equation 3.15, the probability that an edge between SU$_i$ and SU$_j$ can be actually established is equal to the probability that these two nodes are in each other’s transmission range and have at least one available common spectrum band.
The generic element $E[d_{ij}^c]$ can be expressed as:

$$E[d_{ij}^c] = \begin{cases} 
\sum_{k=1}^{S} E[a_{ik}^c] = n \cdot 1 + h \cdot \pi & \text{if } i = j \\
0 & \text{otherwise}
\end{cases}$$  \hspace{1cm} (3.16)

where:

- $n$ is the number of SU$_i$'s neighbors, in terms of radio visibility, which satisfies $E[a_{ij}^c] = 1$; in this case the probability that an edge between nodes $i$ and $j$ can be actually established is equal to 1 because nodes SU$_i$ and SU$_j$ have always at least one available common spectrum band, regardless of PUs' activities;

- $h$ is the number of SU$_i$'s neighbors, in terms of radio visibility, which satisfies $E[a_{ij}^c] = \pi$; in this case the probability that an edge between nodes $i$ and $j$ can be actually established is equal to $\pi$.

By computing the difference between $E[D^c]$ and $E[A^c]$ I derive the expected value of the $S \times S$ average Laplacian matrix $E[L^c]$, where the generic element is defined as:

$$E[l_{ij}^c] = \begin{cases} 
E[d_{ii}^c] & \text{if } i = j \\
-E[a_{ij}^c] & \text{otherwise}
\end{cases}$$  \hspace{1cm} (3.17)

In accordance with the $E[a_{ij}^c]$ and $E[d_{ij}^c]$ definition I can notice that $E[L^c]$ is symmetric and all its row and column sums are equal to 0, as the matrix $L$: this means that $E[L^c]$ has the same properties of the Laplacian $L$. The main difference between the generic element of $L$ and $E[l_{ij}^c]$ is that the former has a deterministic value, whereas the latter has a probabilistic one and it is a function of PUs activity factors. In fact, $E[l_{ij}^c]$ depends on the behavior of PUs characterized by their activity factors: therefore each value $E[l_{ij}^c]$ is function of the probability $\pi$. Consequently, also the value of the second smallest eigenvalue ($\lambda_2\{E[L^c]\}$) is a function of the same probability:

$$\lambda_2\{E[L^c]\} = \lambda_2(\pi).$$  \hspace{1cm} (3.18)

I named the second smallest eigenvalue of $E[L^c]$ cognitive algebraic connectivity.
3.3 $\lambda_2\{E[\mathcal{L}^c]\}$ and $E[\lambda_2^c]$: accuracy versus computational complexity

In Sections 3.2.2 and 3.2.3 I defined the expected value of cognitive algebraic connectivity $E[\lambda_2^c]$ and its estimate $\lambda_2\{E[\mathcal{L}^c]\}$, and the second of these parameters will be used for routing purposes (Section 4.1). It is to be noticed that in general these two parameters have not the same value. The aim of this Section is twofold: on one hand I want to show that the difference between the two parameters can be very small and, consequently, $\lambda_2\{E[\mathcal{L}^c]\}$ can give an accurate estimation of $E[\lambda_2^c]$. On the other hand I want to highlight the advantage of using of $\lambda_2\{E[\mathcal{L}^c]\}$ instead of $E[\lambda_2^c]$, that consists in a drastic reduction of the computational complexity in terms of operations.

As for the accuracy I point out that:

1. $E[\lambda_2^c]$ and $\lambda_2\{E[\mathcal{L}^c]\}$ have the same behavior as functions of PUs’ activities and as functions of number of SUs;

2. the difference between $E[\lambda_2^c]$ and $\lambda_2\{E[\mathcal{L}^c]\}$ decreases when the activity factors of PUs decreases, while, for high values of PUs’ activity factors, this difference increases when $S$ increases (until it settles on a given value);

3. given two different network topologies $A$ and $B$, if $E[\lambda_2^c](A) < E[\lambda_2^c](B)$ then in a great percentage of cases it happens that $\lambda_2\{E[\mathcal{L}^c]\}(A) < \lambda_2\{E[\mathcal{L}^c]\}(B)$.

As for points 1) and 2) I performed an extensive numerical comparison between $\lambda_2\{E[\mathcal{L}^c]\}$ and $E[\lambda_2^c]$ by computing both values in a large number of topologies. In particular, I generated 200 different topologies with three PUs characterized by an activity factor ranging from 0.3 to 0.8 and $S$ SUs ($S = 20, 30, 40, 50, 60, 70$). For each of these topologies I calculated $\lambda_2\{E[\mathcal{L}^c]\}$, $E[\lambda_2^c]$ and the percentage difference between the two values defined as:

$$\delta(\%) = \frac{\lambda_2\{E[\mathcal{L}^c]\} - E[\lambda_2^c]}{E[\lambda_2^c]} * 100$$ (3.19)

Then I averaged the results related to the 200 topologies, obtaining $E[\lambda_2^c]_{\text{mean}}$, $\lambda_2\{E[\mathcal{L}^c]\}_{\text{mean}}$ and $\delta_{\text{mean}}(\%)$. 
Figure 3.6: $E[\lambda_2^{\text{mean}}]$ and $\lambda_2\{E[L^c]\}^{\text{mean}}$ as a function of activity factor of PUs, for different values of $S$.

In Figure 3.6 there are represented $E[\lambda_2^{\text{mean}}]$ and $\lambda_2\{E[L^c]\}^{\text{mean}}$ as a function of activity factor of the three PUs for different value of $S$, averaged on 200 different topologies. It is possible to observe that these two variables decrease when the PUs’ activity factors increase and increase when $S$ increases. In particular, an increment of $S$ equal to 10 ($\Delta S = 10$) entails an increment of $E[\lambda_2^{\text{mean}}]$ and of $\lambda_2\{E[L^c]\}^{\text{mean}}$ that is greater for a high value of $S$. I can state that $E[\lambda_2^{\text{mean}}]$ and $\lambda_2\{E[L^c]\}^{\text{mean}}$ have the same behavior as a function of the PUs activities and as a function of $S$. This means that both are able to capture the impact of PUs’ behavior and number of SUs on network connectivity. The main difference between these two parameters is that, when the activity factors of PUs increases, $\lambda_2\{E[L^c]\}^{\text{mean}}$ decreases slower than $E[\lambda_2^{\text{mean}}]$, i.e. the difference between $E[\lambda_2^{\text{mean}}]$ and $\lambda_2\{E[L^c]\}^{\text{mean}}$ increases when the activity factors of PUs increase. This behavior is confirmed by Figure 3.7, where it is shown the mean percentage difference $\delta^{\text{mean}}(\%)$ as a function of SUs’ number for different value of PUs’ activity factors, averaged on 200 different topologies. It is possible to notice that $\delta^{\text{mean}}(\%)$ increases when PUs’ activity factors increase and, for high values of PUs’ activity factors, this difference increases when $S$ increases until it settles on a given value. In particular, if PUs’ activity factors are lower than or equal to 0.5, $\delta^{\text{mean}}(\%)$ does not reach 10%, when the PUs’ activity factors are equal to
0.6, $\delta_{\text{mean}}$ (%) is lower than 15%, instead for PUs’ activity factors equal to 0.7 $\delta_{\text{mean}}$ (%) does not reach 25% and finally for PUs’ activity factors equal to 0.8 $\delta_{\text{mean}}$ (%) does not reach 35%. I can conclude that $\lambda_2\{E[L_c]\}_{\text{mean}}$ represents an accurate estimation of $E[\lambda_2^c]_{\text{mean}}$ when the PU activities are below 0.6. Since in our practical scenario I am more interested in situations when PUs’ activity factors are quite low (the FCC estimates that the average utilization of licensed bands varies between 15-85% [10]), I can rely on the fact that the percentage difference between these two parameters is very low. Besides, I use $\lambda_2\{E[L_c]\}$ instead of $E[\lambda_2^c]$ with the aim of designing routing solution in cognitive scenarios. However, when surrounding primary radio user are highly active, that is when the activity of PUs on the licensed bands are high (greater than 0.6), a quite stable availability of such frequency bands for a whole communication duration becomes an unrealistic assumption. Consequently, in this case, it is not convenient to design and establish a route for the whole flow, but a more reasonable solution is to opportunistically forward packets without pre-establish a route [11]. The result is that the proposed routing, described in (Section 4.1), is suitable only when the activity of PUs is quite low (from 0.1 to 0.6), that is the case when our approximation ($\lambda_2\{E[L_c]\}$ instead of $E[\lambda_2^c]$) is really correct.

In Section 4.1 I will use $\lambda_2\{E[L_c]\}$ instead of $E[\lambda_2^c]$ in order to compare the
stability and the connectivity of different paths from a source to a destination, by calculating the value of $\lambda_2\{E[L^c]\}$ of each virtual graph associated to each possible path (the concept of virtual graph will be explained in Section 4.1). This is equivalent to compare two different topologies by means of $\lambda_2\{E[L^c]\}$. From this consideration I can state that I am not interested in the absolute value of $\lambda_2\{E[L^c]\}$ compared with the one of $E[\lambda_2^c]$, but it is important that if $E[\lambda_2^c]$ calculated for a given topology $A$ is lower than $E[\lambda_2^c]$ calculated for a different topology $B$, then it should happen that $\lambda_2\{E[L^c]\}$ calculated for $A$ continues to be lower than $\lambda_2\{E[L^c]\}$ calculated for the topology $B$, that is:

$$E[\lambda_2^c](A) < E[\lambda_2^c](B) \Rightarrow \lambda_2\{E[L^c]\}(A) < \lambda_2\{E[L^c]\}(B)$$  \hspace{1cm} (3.20)

I performed a high number (1000) of numerical tests in order to verify the truth of the condition (3.20). Each test consists in creating two cognitive scenarios: both of these two topologies have three PUs, but they have two different values of PUS’ activity factors, ranging from 0.1 and 0.9, and two different number of SUs ($S = 20, 30, 40, 50, 60, 70$). Then for these two topologies I calculated $\lambda_2\{E[L^c]\}$ and $E[\lambda_2^c]$ and tested out the condition (3.20). The final results is that the condition 3.20 is verified 965 times over 1000, that is it is verified with high probability (96.5%). Moreover, it is possible to point out that in the most of the cases the condition (3.20) is not satisfied when PUs’ activity factors are higher than 0.6. In order to prove it I repeated the 1000 numerical tests by considering the first time PUs’ activity factors ranging from 0.1 and 0.6 and the second time PUS’ activity factors ranging from 0.7 and 0.9: in the first case the condition 3.20 is verified 990 times over 1000, whereas in the second case it is satisfied 921 times over 1000. Consequently, in the cases I am interested in, the connectivity of two different topologies can be correctly compared by using $\lambda_2\{E[L^c]\}$ instead of $E[\lambda_2^c]$ with a very high probability (99%).

As for the computational complexity, I want to pay a close attention to the high gain that is obtained if $\lambda_2\{E[L^c]\}$ is used instead of $E[\lambda_2^c]$. The advantage of using of $\lambda_2\{E[L^c]\}$ is that I calculate only one average cognitive Laplacian matrix and its second smallest eigenvalue instead of $2^P$ cognitive Laplacian matrices and their second smallest eigenvalues. In this way the
computational complexity dramatically reduces. A well known method for the eigenvalues computation is the QR method, that is based on matrix factorization. The computational complexity of this method, when a $n \times n$ matrix is considered, scales as $O(n^2)$. Our aim is to calculate the eigenvalues of the Laplacian matrix associated to a virtual graph: as will be said in Section 4.2 a virtual graph is composed by $R = (H + 1) \cdot P$ nodes, where $H$ is the number of hops in the path. Therefore, the computational complexity for the calculation of $\lambda_2\{E[L^c]\}$ scales as $O(R^2)$ instead of $O(2^P \cdot R^2)$ that is the computational complexity required for $E[\lambda_5^2]$.

In Figures 3.8 there are represented the computational complexities for $E[\lambda_5^2]$ and $\lambda_2\{E[L^c]\}$ calculation, as a function of number of PUs, for different values of $H$ (in these two figures the ordinate axes are logarithmically scaled). Obviously the computational complexity for the calculation of $E[\lambda_5^2]$ is always higher than the one required for the calculation of $\lambda_2\{E[L^c]\}$. Besides it it possible to notice that, when the number of PUs increases, the computational complexity for the calculation of $E[\lambda_5^2]$ increases faster than the one required for the calculation of $\lambda_2\{E[L^c]\}$: this means that the gain obtain in terms of computational complexity increases with the number of PUs. This behavior is depicted in Figure 3.9 that represents the percentage gain in terms of computational complexity as a function of number of PUs: I highlight that
the percentage gain is very high even if the number of PUs is low. For example, the percentage gain reaches 85% when $P = 3$ and overcomes 95% when $P = 5$. I can conclude that the use of $\lambda_2 \{E[L^c]\}$ instead of $E[\lambda_2^c]$ is a very good trade off between the accuracy and the computational complexity, especially when PUs’ activity factors are not greater than 0.6.

## 3.4 Performance evaluation: on the behavior of $\lambda_2 \{E[L^c]\}$

In the previous section I showed that in the scenarios I am interested in, it is possible to use the parameter $\lambda_2 \{E[L^c]\}$ with the aim of analyzing the connectivity of secondary networks. In this Section I tested the mathematical framework proposed in Section 3.2.3, by presenting the behavior of $\lambda_2 \{E[L^c]\}$ in some network topologies. The aim is to derive the trend of $\lambda_2 \{E[L^c]\}$ as function of the probability $\pi$, that as explained in Section 3.2.3 is the probability that at least one PU is inactive, when some characteristics of the network scenario vary. In particular, I considered how the variation of i) transmission range of SUs, ii) transmission range of PUs, and iii) number of SUs, impact on $\lambda_2 \{E[L^c]\}$.

I first performed the analysis of the impact of transmission range of the SUs on $\lambda_2 \{E[L^c]\}$, by considering the three scenarios represented in Figure 3.9.
CHAPTER 3. COGNITIVE ALGEBRAIC CONNECTIVITY

Figure 3.10: Three different network scenarios with 3 PUs having transmission ranges equal to 35 meters, 8 SUs and three values of secondary users transmission range (a): $r_{SU}=28$ meters, (b): $r_{SU}=36.5$ meters, (c): $r_{SU}=40$ meters

3.10, where it is assumed that SUs have the same transmission range, indicated with $r_{SU}$, and the same holds for transmission range of PUs, indicated with $r_{PU}$. Three PUs are placed in an area of $100 \times 100$ meters and eight SUs are scattered in this area. $r_{PU}$ is 35 meters, generating CAs represented by circles in Figure 3.10. The reason behind these reduced PU CAs is that I would test scenarios where the transmission of a SU may affect near PU receivers and where SUs are affected by different PUs. Besides, in this figure, links between SUs represent radio visibility relationships. In Figure 3.11 it is shown the behavior of $\lambda_2\{E[L^c]\}$ as a function of the $\pi$ probability in the three scenarios of Figure 3.10. As expected, in all the three cases, the value of $\lambda_2\{E[L^c]\}$ increases when the probability $\pi$ increases. From this result I can infer that the PUs’ activity impacts on degree connectivity and consequently on routing performance in terms of reliability. Moreover, for a given value of the probability $\pi$, $\lambda_2\{E[L^c]\}$ improves when the transmission range of SUs increases. It is possible to notice that for $r_{SU}$ equal to 28 or 36.5 meters the secondary network is at most 1-connected, also in the best case ($\pi = 1$). Instead, with $r_{SU} = 40$ meters, if $\pi > 0.35$ the secondary network becomes 2-connected. This analysis can be used to find the minimum probability $\pi$ (that is the maximum value of PUs’ activities) that allows to obtain the desired value of $\lambda_2\{E[L^c]\}$. In this work I did not consider mutual interference
among SUs and consequently I allowed that two interfering SUs use the same channel. As a consequence, when the transmission range increases the degree connectivity increases too, but also the mutual interference among SUs increases.

In Figure 3.12 it is shown the behavior of the $\lambda_2\{E[L^c]\}$ as a function of the $\pi$ probability in Scenario of Figure 3.10(b), for three different values of $r_{PU}$. In this situation, for a given value of $\pi$, $\lambda_2\{E[L^c]\}$ increases when $r_{PU}$ decreases, since the lower is $r_{PU}$, the smaller is the number of SUs influenced by PUs. I observe that the potential algebraic connectivity, calculated without considering PUs, corresponds to the value of $\lambda_2\{E[L^c]\}$ when $\pi = 1$, that is it is equal to 0.713. The value of $\pi$ such that $\lambda_2\{E[L^c]\}$ is equal to the potential algebraic connectivity is influenced by $r_{PU}$. In fact, the lower is $r_{PU}$, the lower is the probability $\pi$ necessary to obtain the potential algebraic connectivity, that is values of PUs’ activity factors may be higher without compromising $\lambda_2\{E[L^c]\}$. From the Figure 3.12 it is possible to notice that when $r_{PU}$ is equal to 25 meters the potential algebraic connectivity is reached for $\pi = 0.37$, instead when $r_{PU}$ is equal to 40 meters the potential algebraic connectivity is reached only for $\pi = 1$.

In Figure 3.13 it is represented the tree-dimensional behavior of $\lambda_2\{E[L^c]\}$ as function of the probability $\pi$ and of the number of SUs in the network.
I considered a network scenario with 3 PUs and $S$ SUs ($S = 5, 10, 15, 20$) scattered in an area of $100 \times 100$ meters. The $CA$ radius of PUs is 40 meters while the transmission range of the SUs is 45 meters. For each value of $S$ I randomly generated 10 topologies analyzed by means of $\lambda_2\{E[L^c]\}$ and averaged these results. From this figure I can see that $\lambda_2\{E[L^c]\}$ increases when $\pi$ and the number of $SU$s increase. Besides, I can argue that the increase of $\lambda_2\{E[L^c]\}$ related to an increment of $\pi$ is bigger than the increase of $\lambda_2\{E[L^c]\}$ related to the same increment of $S$. The result is that the $\lambda_2\{E[L^c]\}$ is mainly limited by the impact of PUs rather than the number of SUs composing the secondary network. Consequently, it is harder to play with the number of SUs to achieve a given value of $\lambda_2\{E[L^c]\}$ rather than operating on the activity factors of PUs.

To sum up, in this Section I showed that I show that both the extension of the Coverage Area of the PUs and the transmission range of the SUs have an impact on the connectivity. Moreover, I derived that to achieve a given connectivity target in the secondary network, it is more effective to play, when possible, with the activity factors of the PUs rather than with the number of secondary nodes. This results may be used to plan a secondary cognitive radio network on the basis of both the topology of the primary one and the mean activity factors of the licensed users. Furthermore, the
Figure 3.13: Three-dimensional behavior of $\lambda_2\{E[L^c]\}$ as function of the probability $\pi$ and of the number of SUs

The proposed methodology can find applications in the field of the cognitive radio routing to determine the number of primary-free paths in a networks, as well as to characterize the stability of a path as a function of the primary users activities. As for this latter aspect I already started a work that has been presented in [12].
Bibliography


Chapter 4

Gymkhana Routing

4.1 The routing framework

The idea behind the proposed routing scheme is to use $\lambda_2\{E[L^c]\}$ as metric for capturing and comparing the connectivity of different paths, their lengths and their stability in presence of PUs. As shown in Section 3.3 there is a tradeoff between the use of $\lambda_2\{E[L^c]\}$ instead of $E[\lambda_2^c]$. The computational complexity in the use of the former parameter is dramatically reduced at the cost of a loss of accuracy in the estimation of the cognitive algebraic connectivity in case of PUs with high activity factors. However, as will be clear in the following, my goal is to have a metric to compare different paths. I will show with simply but effective examples that the $\lambda_2\{E[L^c]\}$ archives effectively this goal.

4.2 Routing metric: from a chain to a virtual graph

To better understand the relationship of $\lambda_2\{E[L^c]\}$ with the path length and with the PUs’ activity I present simple examples of network topologies where SUs (represented with a circle) are arranged in a chain and a unidirectional data transmission is established between the first node (A) and the last one (D or E). The first two topologies of Figure 4.1 are not influenced by PUs and
Figure 4.1: Six different chains with different PUs influences

have a different number of hops (3 and 4, respectively). The third chain is instead influenced by one PU (with activity factor $a_1 = 0, 4$) on one SU, while the fourth chain contains two adjacent SUs affected by the same PU (with activity factor $a_1 = 0, 4$). Finally, the fifth chain has two adjacent nodes affected by two different PUs with activity factors $a_1 = 0, 4$ and $a_2 = 0, 4$, respectively and the sixth chain has two not adjacent nodes affected by two PUs with activity factors $a_1 = 0, 4$ and $a_2 = 0, 4$, respectively.

By computing $\lambda_2\{E[L^c]\}$ for these six topologies I obtain the results of Table 4.1. It is possible to notice that, when nodes are arranged in a chain, $\lambda_2\{E[L^c]\}$ decreases as the number of hops increases, and with the same number of hops it decreases as the number of nodes affected by PUs increases. Moreover, $\lambda_2\{E[L^c]\}$ presents a greater value for chains where the PUs affecting nodes are on adjacent nodes (chains 4 and 5) with respect to chains where the PUs are on not adjacent nodes (chain 6). In the former case in fact, the number of links affected by the PUs is lower, so the probability that a transmission from the SU at the head of the chain to the SU at the end of the chain succeeds is higher.

The main result is that $\lambda_2\{E[L^c]\}$ embraces in a unique value different characteristics of chains (paths): i) the number of nodes (hops) in the chain; ii) the number of PUs affecting the chain; iii) the activity factors of the PUs; iv) the position of the PUs. However, it is important to point out that the
Table 4.1: Cognitive algebraic connectivity of chains of Fig. 4.1

<table>
<thead>
<tr>
<th>Topology</th>
<th>$\lambda_2{E[L^c]}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chain 1</td>
<td>0.5858</td>
</tr>
<tr>
<td>Chain 2</td>
<td>0.3820</td>
</tr>
<tr>
<td>Chain 3</td>
<td>0.2808</td>
</tr>
<tr>
<td>Chain 4</td>
<td>0.2414</td>
</tr>
<tr>
<td>Chain 5</td>
<td>0.2414</td>
</tr>
<tr>
<td>Chain 6</td>
<td>0.2292</td>
</tr>
</tbody>
</table>

parameter $\lambda_2\{E[L^c]\}$ evaluated on a chain is not able to capture the effect that the switching from one spectrum band to another in a path can have. I can imagine that for routing purposes it is preferable a path using the same spectrum band for all nodes with respect to a path that “jumps” from a spectrum band to another: each time that a node has to perform a switch from a channel to another, it spends time due to hardware issues. In the latter case I have to consider in the path evaluation the effect of switching from a spectrum band to another: I name it switching penalty. To have an example let us consider the two paths represented by chains 4 and 5 where nodes are able to use two SBs (SB$_1$ and SB$_2$). The two chains have the same number of hops and the same PU influence in terms of activity factors, however the switching penalty should be higher for chain 5 than for chain 4. In chain 4 if the spectrum band chosen for the transmission is SB$_2$, the path is crossed without switching from a spectrum band to another. Instead, if SB$_1$ is chosen, when PU$_1$ becomes active nodes B and C should change spectrum band to guarantee a continuity in their transmission. On the contrary, in chain 5 both choices of the initial spectrum band can give rise to a switching to guarantee a continuity in the transmission. This does not emerge from results of Table 4.1 where I can notice that $\lambda_2\{E[L^c]\}$ is the same for both chains 4 and 5.

For this reason, I extended the representation of a path, introducing a new structure associated to it, named virtual graph, that is able to take into account the switching penalty. A virtual graph $V_l$ associated to a path $l$ is defined as a virtualization of the chain representing the path $l$. I associate
CHAPTER 4. GYMKHANA ROUTING

$P$ virtual nodes to each node $i$ of the path $l$. The virtual node $v_i^j(p)$ is the virtualization of node $i$ on the spectrum band $SB_p$ and represents the fact that the node is associated to the $SB_p$ for its transmissions. As a consequence in $V_l$ each node in the path is replaced with the $P$ associated virtual nodes. The number of nodes of $V_l$ is equal to $\mathcal{M}_l = (H_l + 1) \cdot P$ where $H_l$ is the number of hops in the path $l$.

Between two virtual nodes there exists an edge only if these two nodes represent consecutive nodes in the path $l$ or if these two nodes are the virtualizations of the same node on different spectrum bands. In $V_l$ I can distinguish two different kinds of edges: horizontal and vertical edges. The number of horizontal edge is equal to $P \cdot H_l$ and the number of vertical edges is equal to $(\sum_{z=1}^{P-1} z) \cdot (H_l + 1)$. The horizontal edge between $v_i^j(p)$ and $v_{i+1}^j(p)$ indicates that node $i$ receives in the spectrum band $SB_p$ and re-transmits to node $i+1$ over the same spectrum band (with $p = 1, ..., P$), where the couple $v_i^j(p)$ and $v_{i+1}^j(p)$ ($i = 1, 2, ..., H_l$) is the virtualization on the spectrum band $SB_p$ of two consecutive nodes in $V_l$. Besides, each of these horizontal edges has an associated weight (that belongs to range $[0 - 1]$) resulting from the impact of PUs on nodes composing the path (see Section 4.5.1 for these weights calculation). The vertical edge between the virtual nodes $v_i^j(p)$ and $v_i^j(q)$ indicates that node $i$ switches from $SB_p$ to $SB_q$, (with $i = 1, 2, ..., H_l + 1$).

As said before, I extended the representation of a path, from a chain to a virtual graph, because the latter representation is able to penalize a path where it is more probable to have several switchings, differently from the chain representation. Figure 4.2 shows again chains 4 and 5 of Figure 4.1. This time also the virtual graph representations are shown; weights for the links of the virtual graphs are derived in accordance to rules described in Section 4.5.1. If the chain representation is used (cases a) and c)), $\lambda_2\{E[L_c]\}$ has the same value for both chains; instead, if the virtual graph representation is used (cases b) and d)) the value of $\lambda_2\{E[L_c]\}$ related to path$_2$ is lower than the one concerning path$_1$. This penalizes path$_2$ compared to path$_1$. 
4.3 Building blocks of Gymkhana

I formulated my routing framework on the basis of the virtual graph model and on the possibility to measure its cognitive algebraic connectivity via $\lambda_2\{E[\mathbb{L}_c]\}$. I assume that a route discovery phase is performed by using a common control channel, as typically assumed in the literature (e.g., [1], [2], [3]). In fact, the problem of conveying control information without an assigned common control channel is interesting by itself, but is beyond the scope of this thesis. A paper that addresses this issue is [4], where the authors, by exploiting time and space varying spectrum opportunities and by proposing a cluster-based architecture, solve the problem of dynamic assignment of coordination/control channels in CRNs. The route discovery phase allows to collect some key parameters related to candidate paths from an origin (S) to a destination (D). In this way a destination has a path level view where nodes composing the path and activities of the PUs affecting this path are represented. Thanks to the virtual graph representation for each path I model the different spectrum bands available on the path and their possible interconnections. I then use the methodology of Section 4.2 to provide a comprehensive measure of the connectivity of the path.
The resulting routing scheme is called Gymkhana and is based on two main components:

- collection the path level view of all possible paths from a source towards a destination by using an AODV-style mechanism;
- evaluation, at the destination, of the degree connectivity of the virtual graphs, by using the second smallest eigenvalue of the average cognitive Laplacian.

In the following I describe these two parts of the Gymkhana framework.

4.4 Distributed information collection at SUs

The mechanism for collecting the information required for running Gymkhana can be a distributed protocol based on AODV. In this section I briefly describe how an extended version of AODV can be used to this end. I do not enter into details on the implementation of this AODV like approach, since it can be similar to other existing proposals as in [5]. The protocol can be based on the assumption that each SU is able to have the information on which are PUs that can influence its transmission and which are the relevant activity factors. In general this information can be acquired via different procedures. An SU should know if it is in the coverage area of a PU and could measure via the spectrum sensing the activity factor of this PU [6]. If the SU is not in the coverage areas of PUs it should know its position with respect to the PTs in order to verify if it could influence the relative PUs by using Equation 3.7; this can be acquired with distributed procedures (e.g, [7]) or via the access to spectrum maps.

Furthermore, it is important to notice that, when a SU senses the different spectrum bands, it is able to detect an energy that is not only determined by the relative PUs, but also by other SUs that are already using that spectrum bands. In this way I take into account also the activities of other SUs, in the way that the selected path is the one less affected by both PUs and SUs. In this way I am able both to respect the cognitive paradigm, that imposes that a SU can not generate interference to PUs, and to include in my model
the interference determined by other active SUs.

On the basis of the above-mentioned considerations, each SU can compose an interference vector $I^s$ of $P$ elements, where the $p^{th}$ element $\nu_{p,s}$ is composed of two contributions: the one determined by the PU activity and the other due to cumulative activity of other SUs that are already transmitting on the spectrum band $SB_p$.

A source node $S$ broadcasts a route request (RREQ) packet across the network to discover all possible paths towards the destination D (let $L$ be the number of these possible paths). The RREQ arriving at $D$ for the $l^{th}$ path, with $l = 1, \ldots, L$, contains two lists of $H_l + 1$ elements (where $H_l$ is the number of hops in the $l^{th}$ path):

- the list $L^a_l = \{ID_l^i\}$ of the nodes encountered in the path $l$; each node is identified with its unique ID;
- the list $L^b_l = \{I^i_l\}$ of interference vectors of nodes encountered in the path;

with $i = 1, 2, \ldots, H_l + 1$, $ID_1^l = S$ and $ID_{H_l+1}^l = D$. The list $L^b_l$ of interference vectors can be represented as a $P \times (H_l + 1)$ matrix where the generic column is the interference vector $I^i_l = I^i_l^n$, for $i = 1, 2, \ldots, H_l + 1$. Both lists are composed hop by hop. The list $L^a_l$ is initialized with the identifier of node $S$, while $L^b_l$ is initialized with the interference vector $I^S_l$. Each SU receiving the RREQ, checks if its ID is already contained in $L^a_l$. If this is the case, the node discards the RREQ to avoid cycles, otherwise it adds its ID in $L^a_l$ and its interference vector $I^i_l$ in in $L^b_l$ and then broadcasts the RREQ. In this way, the destination node D receives as many RREQs as the possible paths from $S$ towards $D$. Note that unlike the traditional AODV, this routing protocol allows multiple paths to propagate to the destination (as in [5]). Moreover, Gymkhana natively captures the changes that may arise due to mobility and consequently offers quick adaptation to dynamic link conditions, and responds, in a timely manner, to changes in network topology due to node mobility. This is true if the time needed for a SU to change position is greater than the time needed to execute the RREQ and RREP (Route Reply) procedures.

A representation of the main operations performed at a generic SU in a
Figure 4.3: Gymkhana operations performed at a generic SU

Figure 4.4: Example of information collection in Gymkhana for a generic path supposing that no other SUs are active. The element in the interference vectors contains only the contributions due to PUs.

path is reported in Figure 4.3 while in Figure 4.4 it is shown an example of information collection in Gymkhana for a generic path, supposing that no other SUs are active (the element in the interference vectors contains only the contributions due to PUs).

4.5 Path selection

The destination D uses the information contained in the received RREQs to classify paths on the basis of their connectivity and to select one path. The path selection is sent back with a RREP packet on the reverse path towards
The elaboration of the contents of the received RREQs at a destination is done in four steps (right part of Figure 4.5):

1. formation of a virtual graph $V_l$ for each of the $L$ paths;
2. calculation of the second eigenvalue of the Laplacian of the virtual graph of step 1;
3. computation of the utility function associated to the virtual graph $V_l$;
4. comparison of the utility functions of all the received paths and selection of the best one.

### 4.5.1 Formation of virtual graph of a path

This step of the Gymkhana procedure forms the virtual graphs associated to all the paths arrived to the destination nodes via RREQs. For the generic path $l$, $V_l$ can be easily formed on the basis of the list $L_i^l$. It has the structure of a basic trellis repeated a number of times equal to $H_l + 1$.

Weights of horizontal edges are computed on the basis of the elements in $L_i^l$. Let $w_h = w_l[v_i^p(p), v_i^{i+1}(p)]$ be the weight of the edge between $v_i^p(p)$ and $v_i^{i+1}(p)$, for $i = 1, 2, ..., H_l$ and $p = 1, 2, ..., P$. By denoting with $(i_{p,i})$ the $p^{th}$ element of the interference vector $I_i^l$ for the node $i$ in the path $l$, the weight $w_h$ is computed as:
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\[ w_h = \begin{cases} 
1 & \text{if } (\mu_{pi})_l = 0 \land (\mu_{p,i+1})_l = 0 \\
1 - (\mu_{pi})_l & \text{if } (\mu_{pi})_l \neq 0 \land (\mu_{p,i+1})_l \neq 0 \\
1 - [(\mu_{pi})_l + (\mu_{p,i+1})_l] & \text{otherwise}
\end{cases} \] 

From this equation it can be noticed that the horizontal weights depend on PUs and SUs activity. With respect to a full connection (weight=1) of two adjacent nodes not influenced by PUs and by other SUs, the weight is scaled down on the basis of the activities reported in \( L^b_i \).

The weight of a vertical edge \( w_v = w_l[v^i_j(p), v^i_j(q)] \) between \( v^i_j(p) \) and \( v^i_j(q) \), for \( i = 1, 2, \ldots, H_l + 1 \) and \( p, q = 1, \ldots, P \) with \( p \neq q \) is:

\[ w_v = 1 \] 

The vertical edges model the fact that a generic node in the path can perform a switch from a spectrum band to another. These vertical edges can be used when the spectrum band currently used is no more available due to the activation of the relevant PU. This means that the higher is the activity factor of PU that impacts a node in the path, the higher is the probability that this node has to cross a vertical edge to vacate the spectrum band no more available. Obviously a path crossing different spectrum bands is more costly with respect to a path on the same spectrum band. By introducing these links in the virtual graph I can differentiate paths also considering the switching penalty.

4.5.2 Laplacian of the virtual graph

I use the links’ weights of Eq.s 4.1 and 4.2 to build the average cognitive Adjacency matrix \( E[A_i^c] \) and the average cognitive Degree matrix \( E[D_i^c] \) of the virtual graph \( V_i \) (both are \( M_l \times M_l \) matrixes). The generic element of \( E[A_i^c] \) is equal to 0 if it concerns two nodes of the virtual graph that are not connected by an edge, while it is equal to the weight of the edge in the other cases. Similarly, the degree of the generic node of \( V_i \) is equal to the sum of
the weights of edges incident to that node. The average cognitive Laplacian matrix of $V_l$ is then $E[\mathcal{L}_l^c] = E[\mathcal{D}_l^c] - E[\mathcal{A}_l^c]$.

The metric used to evaluate the connectivity of a path is then the second smallest eigenvalues of $E[\mathcal{L}_l^c]$. In accordance with results of Section 3.2.3, the second smallest eigenvalue of $E[\mathcal{L}_l^c]$ (i.e., $\lambda_2\{E[\mathcal{L}_l^c]\}$), can be used to evaluate the degree of connectivity of the virtual graph $V_l$, that is of the path $l$. Moreover, as discussed with reference to Figure 4.1, this metric is able to give less weight to paths that are longer in terms of hops and “virtually longer” due to the fact that they are affected by PUs and eventually by other active SUs. Besides, if I use the virtual graph to represent the path instead of a simple chain, different concatenations of spectrum bands in a path can be established by crossing vertical edges, that is by hopping (and switching) on different spectrum bands. The $\lambda_2\{E[\mathcal{L}_l^c]\}$ is low for a concatenation of spectrum bands that implies a great number of switchings: in this way a path can be “virtually longer” not only because it is affected by PUs and other active SUs, but also because it requires a high number of switchings from a spectrum band to another.

4.5.3 Computation of the utility function

I then associate to path $l$ a utility function $U_l$ defined as:

$$U_l = \frac{\lambda_2\{E[\mathcal{L}_l^c]\}}{\lambda_2\{\mathcal{L}_l^{clear}\}} \cdot \frac{1}{H_l} \quad (4.3)$$

where $\lambda_2\{\mathcal{L}_l^{clear}\}$ is the algebraic connectivity of $\mathcal{L}_l^c$ when all the primary activity factors are equal to 0, that is when there are not PUs influencing nodes of the path $l$.

It is straightforward to demonstrate that the adopted utility function becomes the classical hop count in case of and Ad-Hoc network when the path is not influenced by PUs.

4.5.4 Path selection

Among the $L$ possible paths the destination node selects the one with the highest value of $U_l$.

This metric contemporary accounts for:
1. the number of hops in a path;

2. the PUs behavior, in terms of activity, along the path;

3. the possible interference generated by the activity of other active SUs;

4. the amount of possible spectrum band switchings along the path.

This selection penalizes paths that are more influenced by PUs and by other active SUs and imply a high number of switchings.

### 4.6 Numerical results

I organize the performance analysis in two parts:

- analysis from a topological perspective, without considering traffic supported in the network;
- analysis considering traffic in the network.

#### 4.6.1 Performance analysis from a topological perspective

In this performance analysis I evaluate Gymkhana in different case studies. I aim at analyzing the proposed framework and the relevant models (i.e., the virtual graph), in order to verify if the most convenient path, taking into account the network topology, the average activity factors of the PUs and the penalty due to the switching from a spectrum band to another, is selected. The analysis is made only from a topological perspective without considering traffic supported in the network. I then analytically derived $\lambda_2 \{E[L_f]\}$ in different case studies and verified my intuition on the Gymkhana behavior. I observe that it is not a goal of this analysis the verification of the Gymkhana protocol (e.g., the AODV-like approach) that will be done in the following Subsection. On the contrary I propose significant case studies build up to practically show the potentialities of our mathematical routing framework.

In Figure 4.6 it is shown a network topology with eleven SUs and four PUs. I use this simple example to demonstrate that the switching penalty is
intrinsically considered in the virtual graph model, and taken into account when Gymkhana selects the most convenient path. In particular, the virtual graph model captures two different aspects concerning the switching penalty:

1. the utility function penalizes a path with potentially more switchings with respect to a path with less switchings;

2. the switching penalty of a path increases as the PU’s activities increase.

In fact, as already said in Section 4.2, the operation of switching requires a certain amount of time, that entails an increase in the end-to-end delay. When the activity factor of a PU is high, the probability of switching is high too: this means that it is very likely that a node has to interrupt its transmission to find another available channel or it has to wait for an available channel if no one is free at that moment. Therefore, when in a path the switching penalty is high, it could be more convenient to select another path with more hops and less switchings.

In the network scenario of Figure 4.6 there are three possible paths from S toward D (path$_1$ = S-A-B-C-D, path$_2$ = S-E-F-G-D and path$_3$ = S-H-I-L-D). These paths have the same number of hops and the same influence in case of equal activity factors of the 4 PUs, but the switching penalty is different in the three cases. In fact, in path$_1$ there are three possibilities of crossing the path without performing a switch from a spectrum band to another, in path$_2$ there are two possibilities, instead there is only one possibility in case of path$_3$. This means that the switching penalty of path$_3$ should be higher than path$_2$, which in turn should be higher than path$_1$. This fact can be observed in Figures 4.7(a) and 4.7(b) where I represented the utility functions of these
three paths when $a_1$ varies ($a_2 = a_3 = a_4 = 0.2$ and $a_2 = a_3 = a_4 = 0.4$, respectively). From the figures it appears that the utility function of the path1 ($U_1$) decreases as $a_1$ increases, while $U_2$ and $U_3$ do not vary, since nodes composing this path are not influenced by PU1. In has to be noticed that $U_2 > U_3$, since path3 presents a higher switching penalty: this result confirms the issue 1) previously mentioned. Besides, in Figure 4.7(a), when $a_1$ is less than 0.23, the selected path is path1, instead when $a_1$ becomes greater than 0.23 the selected path changes from path1 to path2. This result confirms that, in the Gymkhana selection, the changing from path1 to path2 does not happen when $a_1$ becomes greater than 0.2, but for a greater value equal to 0.23. In fact, for $a_1 \in \Delta a_1 = (0.2 - 0.23)$, even if the PU influence over path1 is lightly greater than the one over path2, it is convenient to select the first of these two paths because the lower switching penalty counterbalances the greater PU influence. This aspect is more evident in Figure 4.7(b), where the changing from path1 to path2 does not happen when $a_1$ becomes greater that 0.4, but for $a_1 = 0.5$: for $a_1 \in \Delta a_1 = (0.4 - 0.5)$ it is convenient to select path1. Compared with the result of Figure 4.7(a), in this case the size of the interval $\Delta a_1$ increases, because the overall switching penalty of path2 has increased too, due to the increment of $a_2$, $a_3$ and $a_4$. The latter result confirms the issue 2) mentioned before.
In Figure 4.8 it is shown a network topology with ten SUs affected by three PUs. In this analysis I consider four possible paths from the source towards the destination: path\(_1\)=S-F-H-I-D, path\(_2\)=S-A-B-E-I-D, path\(_3\)=S-A-B-E-D and path\(_4\)=S-G-D. Different situations emerge:

- path\(_1\) and path\(_3\) have the same number of hops, but path\(_1\) should have a higher switching penalty;
- path\(_2\) has one more hop than path\(_1\) and path\(_3\);
- path\(_2\) should have a lower switching penalty than path\(_1\) and the same switching penalty of path\(_3\);
- path\(_4\) has the lowest number of hops compared with the other three paths, but the node G is affected by all PUs.

In Table 4.2 I reported the utility functions of these four paths when \(a_1 = a_2 = a_3 = 0.4\), \(a_1 = a_2 = a_3 = 0.6\) and \(a_1 = a_2 = a_3 = 0.8\), respectively. It is possible to make the following considerations:

1. for all the three activities combinations, \(U_3\) is higher than \(U_1\), since path\(_1\) has a higher switching penalty compared with path\(_3\);

2. for all the three activities combinations, \(U_3\) is higher than \(U_2\), because these two paths have same switching penalty but path\(_2\) has one more hop;
3. for the first two cases of activities combinations, $U_1$ is higher than $U_2$; this means that, for quite low values of activities (between 0.1 and 0.6), the addition of one hop in the path causes a reduction of the utility function higher that the one caused by the switching penalty; besides, the difference between $U_1$ and $U_2$ decreases when the PUs activity factors increase, that is when the switching penalty increases;

4. when $a_1 = a_2 = a_3 = 0.8$, $U_1 < U_2$, since when PUs’ activity factors increase, the switching penalty increases too and, consequently, it is more convenient to cross one more hop rather than to cross a path where several switchings are highly probable;

5. among these four paths, the shortest, path$_4$, is the most convenient only when PUs’ activity factors are low, instead when PUs’ activity factors are comparatively high it is better to select path$_3$.

To better explain points 3) and 4) I derive the expected value of the delivery delay for a packet crossing path$_1$ and path$_2$. I suppose that:

- in each hop the secondary user randomly chooses a spectrum band among the three SBs, the three alternatives are equiprobable;

- $t_{tx}$ is the time needed for transmitting a packet on a single hop (each hop is assumed with the same link capacity);

- on each hop, when the transmitting node is affected by PU$_p$, the spectrum band $SB_p$ is occupied with probability $a_p$ and it is free with probability $(1 - a_p)$;

- the spectrum band is independently selected at each hop;

- $t_{sw}$ is the switching delay and takes into accounts the time needed to vacate the used spectrum band, to search, by means of spectrum sensing, a new free spectrum band and to tune the transmitter in this new spectrum band; this delay is proportional to the number of SBs $P$ (3 in this example).
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Table 4.2: Utility functions of the four paths of Figure 4.8 for different values of activity factors

<table>
<thead>
<tr>
<th>Path</th>
<th>$a_{1,2,3} = 0.4$</th>
<th>$a_{1,2,3} = 0.6$</th>
<th>$a_{1,2,3} = 0.8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U^c$</td>
<td>0.2038</td>
<td>0.1757</td>
<td>0.1426</td>
</tr>
<tr>
<td>$U^c$</td>
<td>0.1802</td>
<td>0.1686</td>
<td>0.1556</td>
</tr>
<tr>
<td>$U^c$</td>
<td>0.2192</td>
<td>0.2021</td>
<td>0.1838</td>
</tr>
<tr>
<td>$U^c$</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
</tr>
</tbody>
</table>

With these assumptions the expected value of the delivery delay of a packet across path 1 and path 2 are $5 \cdot t_{tx} + a_1 \cdot t_{sw}$ and $4 \cdot t_{tx} + 2/3 \cdot (a_2 + a_3) \cdot t_{sw}$, respectively. Therefore, considering that in our example I suppose that $a_1 = a_2 = a_3$, it is more convenient to select a path with one more hop rather than a shorter path with higher switching penalty, if it results $5 \cdot t_{tx} + a_1 \cdot t_{sw} < 4 \cdot t_{tx} + 4/3 \cdot a_1 \cdot t_{sw}$, that is if $t_{tx}/t_{sw} < a_1/3$. By assuming some appropriate numerical values for the above parameters in a cognitive radio network: i) a time for sensing around 25 msec for each single spectrum band [8] that results in a $t_{sw} = 75$ msec for 3 spectrum bands; ii) and a link capacity equal to 200 kbit/sec; iii) a packet size of 4000 bit; the above condition is verified if $a_1 \geq 0.8$. This confirms in this example the result of point 4).

Analogous considerations can be done by observing Figures 4.9(a), 4.9(b) and 4.9(c), where there are represented the utility functions of the four paths previously considered as a function of the activity factor of PU 1 when $a_2 = a_3 = 0.4$, $a_2 = a_3 = 0.6$ and $a_2 = a_3 = 0.8$, respectively. I can notice that in all the three cases the intersection point between path 3 and path 1 (that corresponds to the changing from path 3 to path 1) does not happen for $a_1$ equal to $a_2$ and $a_3$, but for a greater value of $a_1$ (named $a_{1,intersection}$). This is because for $a_1 \in \Delta_{a_1} = (a_2 - a_{1,intersection})$, the higher impact due to PU 1 on path 3 is counterbalanced by the higher switching penalty of path 1. Moreover, the interval $\Delta_{a_1}$ increases when $a_2$ and $a_3$ increases, making the switching more probable. Moreover, it is always true that $U_3$ is higher than $U_2$, since path 2 has one more hop and the same switching penalty. When $a_2 = a_3 = 0.4$, $U_1^c$ is always higher than $U_2^c$ for each value of $a_1$, because it is more convenient to cross a path where switches are not very likely rather
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Figure 4.9: Utility function for path1=S-F-H-I-D, path2=S-A-B-E-I-D, path3=S-A-B-E-D and path4=S-G-D of Figure 4.8, as a function of the activity factor of PU1: (a) $a_2 = a_3 = 0.4$ (b) $a_2 = a_3 = 0.6$ (c) $a_2 = a_3 = 0.8$.

than to cross one more hop in the path. Instead, when $a_2 = a_3 = 0.6$ I can notice that $U_1^c$ is higher than $U_2^c$ only for $a_1 \geq 0.5$, but for $a_1 < 0.5$ there is the opposite situation, since for low values of $a_1$ it is more convenient to cross a path with one more hop characterized by a low PU impact rather than a path where there is a high probability to perform a switch. Instead, when the additional hop is characterized by an high PU activity, it is better to cross a path where switches are probable rather than to be interrupted for a long time on that additional hop. Finally, when $a_2 = a_3 = 0.8$, the utility function of the path2 is always higher than $U_1^c$ for each value of $a_1$, since in this case the switching penalty is very high, that is it is very likely that on path1 many interruption due to switches can happen. It is possible
to notice that when $a_2 = a_3 = 0.4$ the path4 is always the best path for each value of $a_1$. Instead when $a_2 = a_3 = 0.6$ path4 is the best one only for $a_1 \leq 0.57$, instead for $a_1 > 0.57$ the best path becomes path3. Finally, when $a_2 = a_3 = 0.8$ the best path is path3 for each value of $a_1$.

To show in more complex scenarios the behavior of Gymkhana I randomly generated cognitive networks with three PUs and 15 SUs, for three different combinations of $a_1$, $a_2$ and $a_3$. Figure 4.10(a) shows the case of $a_1 = 0.4$ $a_2 = 0.2$ $a_3 = 0.3$, Figure 4.10 (b) shows the case of $a_1 = 0.4$ $a_2 = 0.3$ $a_3 = 0.9$, while Figure 4.10(c) shows the case of $a_1 = 0.7$ $a_2 = 0.9$ $a_3 = 0.8$. The
primary networks are represented in terms of coverage areas, with three different semi-circles. The secondary network is represented with nodes colored in gray-scale depending on the number of PUs that affect each node and on the activity factors of these PUs. The color of a SU is as darker as the number of PUs that may condition its transmission increases, or as the activity factors of these PUs increase. For example, in Figure 4.10(b) nodes influenced by PU\(_2\), having an activity factor equal to 0.9, are darker than the same nodes in Figure 4.10(a) where PU\(_2\) has an activity factor equal to 0.2. A black node indicates that this node is influenced by all the PUs with high activity factors, on the other side a white node indicates that this node can always transmit regardless of the PUs’ activity, since it is not influenced by PUs. A link between two secondary nodes is represented by a segment joining them.

For each of these three cases I considered the transmission from the source 15 toward the destination 5 and evaluated the utility function of all the possible paths between these two nodes, by using the Equation 4.3. Figure 4.10(d) plots the utility function values for 25 possible paths, ordered with an increasing number of hops, in the three different cases; in the three curves the maximum value of the utility function is marked with a red circle.
case (a) the chosen path is $15 - 7 - 9 - 5$ (indicated with dashed lines in Figure 4.10(a)). As can be observed in Figure 4.10(d) the corresponding utility function presents its maximum for the first path that is the shortest one, since there are not other paths crossing less influenced nodes. Instead, in cases (b) and (c) Gymkhana chooses the path $15 - 14 - 12 - 6 - 5$ (indicated with dashed lines in Figures 4.10(b) and 4.10(c)): this path has one more hop compared with path $15 - 7 - 9 - 5$, but it is composed by lighter nodes (that is less influenced nodes).

In Figure 4.11 I compare the utility function of Gymkhana with classical hop count (where the utility is inversely proportional to the number of hops). I again analyzed the topology of Figure 4.10, but the results can be simply generalized. It can be noticed that with the classical hop count paths having the same number of hops have the same utility function, instead with Gymkhana paths with the same number of hops can have different values of utility function depending on the overall impact due to PUs. Consequently, Gymkhana gives the same result of a classical hop count when the paths with different number of hops have the same overall PUs’ influence. Instead, when there exist paths heterogeneous in terms of PUs’ influence, Gymkhana is able to capture this difference and tries to avoid obstacles represented by nodes affected by PUs: in this case the selected path can be different from the one simply based on the hop count. For example, in Figure 4.11 in case of activities $a_1 = 0.7 \ a_2 = 0.1 \ a_3 = 0.6$ Gymkhana chooses the same path of the hop count, while the selected path is another one when the activities are $a_1 = 0.1 \ a_2 = 0.9 \ a_3 = 0.2$; this happens because PU$_2$ does not influence nodes in this path.

4.6.2 Performance analysis in presence of traffic

In this Section I evaluate the performance of Gymkhana when a single traffic relationship is established between a transmitting and a receiving nodes. To this aim I compare the cognitive utility function $U^c$ defined in the Equation 4.3 with other two utility functions:

- a utility function that chooses the path with the minimum number of hops, $U^{hop}$.
• a utility function that only considers the activity of PUs, without taking into account the hop count, $U^{activity}$ (in this case the selected path is the one characterized by the minimum overall activity).

I named the routing scheme based on these latter utility functions $U^{hop}$ and $U^{activity}$, $minHop$ and $minAct$, respectively. As already said, Gymkhana selects the path with the greatest $U^c$ while $minHop$ and $minAct$ select the path with the lowest $U^{hop}$ and $U^{activity}$, respectively.

I compare the performance of these three routing schemes, in terms of packet delivery ratio (PDR) and end-to-end packet delay (end-to-end PD). The first performance parameter is defined as the number of data packets actually received by the destination divided by the number of packets issued by the corresponding source node and multiplied by 100. The end-to-end PD is instead measured as the time elapsing between the transmission of the packet by the source node and its arrival to the destination, averaged on all the packets received by the destination node.

In order to evaluate the performance of the three routing metrics, I generated cognitive scenarios as described in Section 3.1. After the generation of a cognitive topology, this I implemented a tool that:

• derive all possible paths from a given source to a given destination;

• simulate a traffic session in the secondary network which opportunistically accesses the channels when these are available as described in Section 3.1.

The primary transmissions happen according to a Poisson process with given average activity factors for each PU ($a_p$), while the secondary traffic is simulated as CBR traffic between two end-points. The cognitive behavior of the secondary nodes is achieved by periodic sensing operation, performed asynchronously by each node in the path: in this way they dynamically select a channel based on the activity of the PUs. I also assume a collision and error free data link layer. Besides, I suppose that, when a node in the path is highly affected by PUs and does not find any available channel, temporarily stores the packets to be transmitted in a buffer until a channel is free again.
The buffer size is finite. Therefore, when one node has to interrupt its transmission waiting for a free channel, it can happen a buffer overflow resulting in packet losses. Simulations parameters are reported in Table 4.3.

In order to point out differences among Gymkhana, minAct and minHop, I consider one topology, shown in Figure 4.12, where there are 50 SUs and 3 PUs characterized by activity factors equal to 0.7, 0.5 and 0.4 and a buffer size equal to 150 packets. In this network topology I choose source and destination (indicated with Sx and Dx, respectively) and compare the performance of the three paths chosen by these routing schemes. As depicted in Figure 4.12(c), minHop selects the path with the minimum number of hops between Sx and Dx, without taking into account how much nodes in this path are affected by PUs. MinAct uses a diametrically opposite criteria to select the path (depicted in Figure 4.12(b)), since it only tries to avoid nodes highly affected by PUs (node characterized by a dark grey), without considering that this choice could involve a not negligible increment in the number of hops. Instead the path chosen by Gymkhana (see Figure 4.12(a)) allows to obtain a trade-off between low number of hops and low PUs’ activities. In Figure 4.12(d) I reported the PDR and the end-to-end PD of these three paths, showing that the path selected by Gymkhana obtains the best performance. Paths chosen by Gymkhana and by minAct have the same PDR, instead the path selected by minHop has a lower PDR, since nodes crossed in this path are frequently and heavily interrupted by PUs and consequently they have to wait.
for an available free channel for a long time interval, storing a big amount of packets and causing buffer overflows. In the path selected by minHop, the interruption in packets transmission due to the presence of PUs reflects also in a significant increment of the end-to-end PD. Instead the difference, in terms of end-to-end PD, between Gymkhana and minAct is due to the time necessary to cross one more hop in case of path chosen by minAct.

Figure 4.13 shows the performance of paths selected by using the three different routing schemes in 10 different networks randomly generated: in all these networks there are 50 SUs and 3 PUs characterized by different activity factors ranging from 0.2 to 0.9. In all these networks the performance of the
path picked out by Gymkhana overcomes the one of the other two paths. The same conclusions can be outlined by analyzing Figure 4.14, where I reported the performance of the three routing schemes, obtained by averaging them in case of 35 different networks composed of 50 SUs and 3 PUs characterized by different activity factors ranging from 0.2 to 0.9. I can state that $\text{nimHop}$ obtains, on average, the worst performance, Gymkhana achieves, on average, the best performance and $\text{minAct}$ gets, on average, intermediate performance.

Figure 4.15 depicts, for the paths chosen by using the three routing schemes, the end-to-end PD as function of PUs’ average activity factor: it is important to notice that in each network scenario used for this comparison, each PU is characterized, in general, by its own activity factor and I name PUs’ average activity factor their arithmetic mean. In this way it is possible to analyze how the performance varies when the PUs’ average activity factor varies. In all the three cases the end-to-end PD increases when the PUs’ average activity factor increases, but the performance worsening is greater.
in case of minHop rather than in Gymkhana and minAct. Besides, the difference in the end-to-end PD between Gymkhana and minHop increases as the PUs’ average activity factor increases, while the same difference between Gymkhana and minAct remains quite constant. This is because the difference in the end-to-end PD between Gymkhana and minAct is essentially determined by the different number of hops, instead the difference in the end-to-end PD between Gymkhana and minHop is due to the fact that, in case of path selected by minHop, it is extremely probable that before transmitting nodes have to wait for an available free channel, and this waiting time increases as the PUs’ average activity factor increases.

In the results shown until now I suppose that each SU has a buffer of size equal to 150 packets. However, in an opportunistic access, the buffer size has a key impact on the network performance. The effect of the buffer size is evident in the PDR: Figure 4.16 represents, in case of Gymkhana, minAct
and minHop, the PDR as function of PUs’ average activity factor, for three different buffer size (75, 150 and 300). It is possible to notice that in all the three cases, PDRs decrease as the buffer size decreases and as the PUs’ average activity factor increases: in fact, when PUs are highly active and when the buffer stores a low number of packets, there is a high probability of a buffer overflow. However, the performance worsening, in terms of packet loss, is higher for minHop and minAct rather than for Gymkhana. MinHop achieves the lowest PDR, since it is not interested in avoiding highly affected nodes, but only in minimizing the number of hops. As for the comparison between Gymkhana and minAct I can state that, for a single hop, the percentage of lost packets in case of minAct is approximately equal to the one of Gymkhana, since both select a path whose nodes are not heavily affected, but the overall percentage of lost packets is greater in case of minAct rather than in case of Gymkhana since, in the first case, the cumulative loss of packets is obtained summing the contributions of more hops.

Finally, I show that, given an arbitrary network scenario, the path picked up by Gymkhana obtains the best performance when compared with all other possible paths among the chosen couple source-destination. To this aim I randomly generated a network topology of 50 SUs and 3 PUs characterized
by activity factors equal to 0.7, 0.5 and 0.4 and set a source-destination couple. Then I derived for all the paths in the given network the relative $U^c$, by using Eq. 4.3, and compute their PDR, end-to-end PD and number of hops (see Figure 4.17). It is possible to observe that the greatest $U^c$ is obtained for two different paths and consequently Gymkhana can indifferently select one of the two (I circumscribe them with a circle), since they have the same performance. As confirmed in Figure 4.17, the Gymkhana path has the highest PDR and the lowest end-to-end PD. The path with the minimum number of hops has scarce performance; this can be due to the fact that nodes in this path are continuously interrupted by PUS. Besides, there are some paths that have similar performance when compared with Gymkhana path: these are paths that have a low influence of the PUs, but have some more hops than Gymkhana path (in fact these paths have a little bit higher end-to-end PD with respect to Gymkhana path and this can be evident in Figure 4.18 where I zoom the end-to-end PD of Figure 4.17 ).
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Figure 4.17: Comparison, in terms of utility function, PDR, end-to-end PD and hop count, among the path chosen by Gymkhana and all the other possible paths present in a network topology, for a fixed couple of source-destination.

Figure 4.18: Zoom of the end-to-end PDs of Figure 4.17 close to the one of Gymkhana path.
Bibliography


Chapter 5

Conclusions

The work I have realized during my PhD thesis can be divided into two phases. In the first phase I analyzed the connectivity of a CRAHN, by using the second smallest eigenvalue of a revised Laplacian matrix, where the presence of PUs is taken into account; in the second phase the worked up concept of connectivity is exploited to design a routing scheme that aims at routing data packets across paths that avoid obstacles, that are network zones that do not guarantee stable and highly connectivity.

As for the first phase I proposed a mathematical framework based on the algebraic connectivity to model and evaluate the connectivity of CRAHNs. I revised the concept of connectivity to take into account the heterogeneity in the spectrum availability perceived by SUs and I proposed an extension of the Laplacian matrix to include the probabilistic behavior of the PUs. First I provided a method to calculate the expected value of the cognitive algebraic connectivity, since the algebraic connectivity is a function of PUs behavior. Then I propose a methodology that reduces the computational complexity of this average measure and obtains a good estimation of the expected value of cognitive algebraic connectivity: this parameter is the second smallest eigenvalue of the expected value of the Laplacian matrix, that I name cognitive algebraic connectivity. Through numerical examples I showed that the cognitive algebraic connectivity, is quite close to the expected value of the algebraic connectivity. This is particularly true in scenarios where the activity of the primary users is limited to a maximum of 60%. Therefore I
am confident that this model may have a practical value in cognitive radio network scenarios with the benefits of a reduced complexity and attractive performance behaviors. The proposed metric has been used to evaluate the impact of the primary activities on the secondary network, and I showed that both the transmission ranges of PUs and SUs have an impact on the connectivity. Moreover, I derived that to achieve a given connectivity target in the secondary network, it is more effective to play, when possible, with the activity factors of the PUs rather than with the number of SUs. These results may be used to plan a secondary cognitive radio network on the basis of both the topology of the SUs and the mean activity factors of the licensed users.

As for the second phase, I described a new routing scheme for CRAHNs where the cognitive algebraic connectivity is used as metric for capturing and comparing the connectivity of different paths, their lengths and their stability. The proposed utility function measures the connectivity of different paths, by taking into account the PUs behavior, the penalty for spectrum band switching and the hop count. It is able to give a lower weight to routes where nodes are continuously interrupted by PUs transmissions; these paths become longer with respect to paths where nodes have more spectrum availability and are not selected during the traditional routing decision, that is normally based on hop count criteria. Besides I model a single path with a virtual graph that intrinsically takes into account the switching penalty. The performance analysis showed the effective capability of capturing, by performing a unique computation on the average Laplacian matrix, key topological characteristics of network paths. In particular, I tested if the most convenient path, taking into account the network topology, the average activity factors of PUs and the cost due to the switch from a channel to another, is selected. This analysis is made only from a topological perspective without considering traffic supported in the network. Then I considered traffic in the network and I evaluate the performance of the Gymkhana routing in terms of end to end delay and percentage of lost packet. Results showed that the path selected by Gymkhana performs better if compared with the path chosen with hop-count criteria, since Gymkhana captures the heterogeneity due to PU behavior and tries to avoid obstacles represented by nodes affected by
PUs. Moreover, the path selected by Gymkhana performs also better than a path chosen by only considering the activity of PUs, without taking into account the hop count. Therefore, Gymkhana is able to combine the hop count criteria with the minimum PUs activity criteria, assuring the best path selection.
Thanks a lot to everyone who come with me during this way...