

Nonlinear ordinary differential equations: perturbative methods & applications

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Course Description: This Course provides some methods to study nonlinear ordinary differential equations aiming to construct solutions to nonlinear problems which arise in applications in all cases in which a *small* parameter appears. The arguments can be schematically listed in:

- a) Straightforward Perturbation Method;
- b) Multiple Scale Method;
- c) Singular Perturbation Method;
- d) Boundary Layer Method.

Some background notions on Asymptotic Expansions and their application to study ordinary differential equations open the course. Then, the different listed methods are presented and illustrative examples are studied in detail. Critical aspects as well as advantages of each method are pointed to the students' attention. In addition, via computer algebra methods, the solutions of the problems are constructed and plotted. Cauchy and boundary value problems are both treated. As a first *toy problem*, the Cauchy problem in the case of a linear weakly damped oscillator is studied. Then, nonlinear o.d.es, such as Duffing equation, are studied. Also the Van der Pol equation, which can be used to model the cardiac cycle, is analyzed. In most of the provided examples, various methods are applied and a comparison among the different approximations obtained and their related region of validity (in time or space) is given. A variety of examples of application is provided and the students are invited to actively participate developing a personal project.

An overview on how to apply Perturbation Methods in the case of partial differential equations closes the course.

Text(s): Selected Chapters from

- *Introduction to Perturbation Methods*, M.H.Holmes, Introduction to Perturbation Methods, Springer, New York, 1995; **Author(s)**: M.H.Holmes; **ISBN-13**: 978-0000000000
- M. Lo Schiavo: Note di sistemi dinamici, SIMAI e-Lecture Notes, Vol 12 (2013) <http://cab.unime.it/journals/index.php/lecture/article/view/928>, **ISBN-13**:978-88-905708-5-8